CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE

MATH 822: ORDINARY DIFFERENTIAL EQUATIONS

STREAMS: MSC (AGEC) ODEL **TIME: 3 HOURS**

DAY/DATE: TUESDAY 06/04/2021 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS:

QUESTION ONE (20 MARKS)

(a) Explain the meaning of the following

(i) Eigenvalue problem [2 marks]

(ii) Eigenvalue function [2 marks]

(iii) Sturm Lowville problem [2 marks]

(iv) Boundary value problem [2 marks]

(b) For the boundary value problem

$$\frac{d^2y}{dx^2}+\lambda y=0,\ y(o)=y(c)=0$$

Find

(i) Eigenvalues [6 marks]

(ii) Eigenvalue functions [6 marks]

QUESTION TWO (20 MARKS)

(a) State the following

(i) Condition for a linear D.E to be exact [2 marks]

(ii) The condition for a non-linear D. Eg to be exact [2 marks]

(b) By reducing the order, solve the now linear D.E

$x^2 y'' + (2x + xy) y'' + x(y')^2 + 3 y y' = 0$ [16 marks]

QUESTION THREE (20 MARKS)

(a) (i) Show that the functions

$f(x) = -6x + 2 f_2(x) = 6x^2 - 6x + 1 \wedge f(x) = (x-1)$ are mutually orthogonal

[6 marks]

(ii) Find the orthonormal set [4 marks]

(b) Write the Eigenvalue problem as a sturm Lowville problem

$x^2 y'' + x y' + \lambda y = 0, y(1) = 0, y(2) = 0$

$y'' + 3 y' + (\lambda + 2) y = 0\, y(o) = 0, y'^{(1)} = 0$ [4 marks]

(c) Given the general solution of a Bessel n which is $J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(m-n)!} \left(\frac{x}{2}\right)^{2m-n}$

Show that $\frac{d}{dx}\left[x^v J\ (x)\right] = x\ J_{-1}(x)$ [6 marks]

QUESTION FOUR (20 MARKS)

(a) Solve the B.V.P for $\lambda = k^2 > 0$

$\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4\ y + 9\lambda y = 0$

$y(o) = 0, y(1) = 0$ [8 marks]

(b) Show that $\frac{d^3 y}{dx^3} + \left(x + 1 + \frac{3}{x}\right)\frac{d^2 y}{dx^2} + 2\left(2 + \frac{1}{x}\right)\frac{dy}{dx} + \frac{2}{x^2} y = 0$ is exact and hence solve

[7 marks]

(c) Consider the set of functions $\{1, \cos x, \cos 2x \ldots\}$ on the interval $[-\pi, \pi]$. Given that the norm of 1 is $\sqrt{2\pi}$ and norm of $\cos nx = \sqrt{\pi}$. Find the orthonormal set of $\{1, \cos x, \cos 2x \ldots\}$

[3 marks]
