CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATIONS FOR THE AWARD OF MASTERS OF SCIENCE IN APPLIED MATHEMATICS.

MATH 832: METHODS OF APPLEID MATHS II

STREAMS: **TIME: 3 HOURS**

DAY/DATE: WEDNESDAY 7/4/2021 **2.30 PM – 5.30PM**

INSTRUCTIONS:

- Answer ANY three questions

QUESTION ONE

a. Show that [7marks]

$$\sqrt{\left(\frac{1}{2}\pi x\right)}\, J_{\frac{3}{2}}(x) = \frac{\sin x}{x} - \cos x$$

Given that

$$J_n(x) = \frac{x^n}{2^n \overline{|(n+1)}}\left[1 - \frac{x^2}{2.2.(n+1)} + \frac{x^4}{2.4.2^2(n+1)(n+2)} - \ldots\right]$$

Where $J_n(x)$ is the Bessels function of order n.

b. Express $J_4(x)$ in terms of $J_1(x)$ and $J_0(x)$ given that [5marks]

$$J_{n+1} = \frac{2n}{x} J_n - J_{n-1}$$

c. Prove that $J_n(x)$ is the co efficient of z^n in the expansion of [6marks]

$$e^{\frac{x}{2}\left(z - \frac{1}{z}\right)}.$$

d. Prove that [2marks]

$$x\sin x = 2[2^2 J_2 - 4^2 J_4 + 6^2 J_6 - ...]$$

QUESTION TWO

a. Prove that [6marks]

$$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\left(\frac{3-x^2}{x^2}\right)\sin x - \frac{3\cos x}{x}\right]$$

Given that

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$$

Where $J_n(x)$ is the Bessels function of order n.

b. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre Polynomials [6marks]

c. Prove the following, where $P_n(x)$ is the Legendres Polynomial

(i). $P_n(x) = 1$ [4marks]

(ii). $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2-2x}}$ [4marks]

QUESTION THREE

a. Prove that $(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$, where $P_n(x)$ is the Legendres Polynomial [5marks]

b. Solve the integral equation below [5marks]

$$y(x) = x + \lambda\int_0^1 (xz + z^2)y(z)\,dz.$$

c. Find the eigenvalues and corresponding eigenfunctions of the homogeneous Fredholm equation below [5marks]

$$y(x) = \lambda \int_0^{\pi} \sin(x+z)\, y(z)\, dz.$$

d. Use the Neumann series method to solve the integral equation [5marks]

$$y(x) = x + \lambda \int_0^{1} xzy(z)\, dz.$$

QUESTION FOUR

a. Find the curve connecting the points (x_1, y_1) and (x_2, y_2) which when rotated about the x axis gives a minimum surface [8marks]

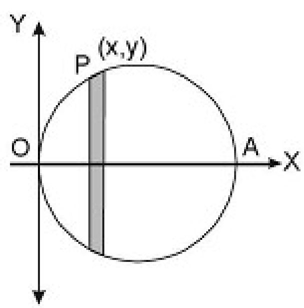
b. Find the shape of the curve of the given perimeter enclosing maximum area [10marks]

c. Solve the homogeneous Fredholm equation below [2marks]

$$y = f + \lambda \mathcal{K} y.$$

QUESTION FIVE

a. Find the solid of maximum volume formed by the revolution of a given surface area as shown below [8marks]



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b. Use Schmidt–Hilbert theory to solve the integral equation [6marks]

$$y(x) = \sin(x+\alpha) + \lambda \int_0^{\pi} \sin(x+z) y(z)\, dz.$$

c. Test for an extremum the functional [6marks]

$$I\left[y(x)\right]=\int_0^1\left(xy+y^2-2y^2y'\right)dx,\ y(0)=1, y(1)=2$$

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