CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

MATH 845: STATISTICAL INFERENCE

STREAMS: MSc. (App stat) **TIME: 3 HOURS**

DAY/DATE: THURSDAY 07/10/2021 **2.30 P.M. – 5.30 P.M.**

INSTRUCTIONS

- **Answer ANY THREE Questions**.
- **Show your working clearly**

QUESTION ONE [20 MARKS]

a) Define the terms given below illustrating with an example

   i) Weakly consistent

   ii) efficiency

   iii) Completeness (6 marks)

b) Let $x_1, x_2, x_3, \ldots, x_n$ be i. i. d Bernoulli (1, p) random variables with p.m.f.

$$f(x,p)=\begin{cases} p(1-p)^{1-x}, x=0,1 \\ 0, otherwise \end{cases}$$

Find the Uniformly Most Powerful size $\propto$ test for $H_0: p=p_0$ against $H_1: p=p_1$ where $p_1 > p_0$ (10 marks)

c) Suppose that $x_1, x_2, x_3, \ldots, x_n$ $i.i.d\, Bernoulli(p)$

Find the maximum likelihood estimator (MLE) for p (4 marks)

QUESTION TWO [20 MARKS]

Let $y_1, y_2, y_3, \ldots, y_n$ be a random sample from density:

$$g(y, \beta) = \begin{cases} \frac{\beta^3}{2} y^2 e^{-\beta y}, y > 0, \beta > 0 \\ 0, otherwise \end{cases}$$

i) Show that this distribution belongs to the exponential family of densities and hence find a sufficient statistic for β

ii) Find the MLE for β and show that it is consistent

iii) Is the estimator obtained in part (ii) MVUBUE? Give reasons for your answer.

QUESTION THREE [20 MARKS]

a) Let $x_1, x_2, x_3, \ldots, x_n \ \ i.i.d\, N(\mu, \sigma^2)$ random variables. Find the Likelihood Ratio Test (LRT) for $H_0: \mu = \mu_0$ against $H_0: \mu \neq \mu_0$ and in a case where σ^2 is unspecified.

(10 marks)

b) Consider a random sample of n independent random variables $X_1, X_2, X_3, \ldots, X_n$ be i. i. d B (1, δ) with p.m.f.

$$f(x, \delta) = \begin{cases} \delta^x (1-\delta)^{1-x}, x = 0,1 \\ 0, otherwise \end{cases}$$

Find a minimal sufficient statistic for δ. (10 marks)

QUESTION FOUR [20 MARKS]

Let $y_1, y_2, y_3, \ldots, y_n \ \ i.i.d\, N(\mu, \sigma^2)$

a) Show that $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$ is asymptotically biased estimator of σ^2. How would you fix the bias? (5 marks)

b) Show that the sample mean is a UMVUE for μ (7 marks)

c) Obtain the asymptotically efficient MLE for unbiased estimator of σ^2

(8 marks)

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