CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF SCIENCE

PHYS 437: QUANTUM MECHANICS II **TIME: 2 HRS**

Streams: BED (SCI) and BSC **DAY/DATE:………………………**

INSTRUCTIONS:

- Answer Question One in Section A and any other Two Questions in Section B
- Do not write anything on the question paper
- This is a closed book exam, No reference materials are allowed in the examination room
- There will be No use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely
- **Useful information and constants**: Rydberg's constant**:** 1.9074 x 10^7 m^{-1}

QUESTION ONE (30 MARKS)

a) State the Pauli's exclusion principle. **(1 Marks)**

b) Define the following terms; **(4 Marks)**

(i) Normal Zeeman effect

(ii) Stalk effect

c) State the variation principle. **(2 marks)**

d)

i. Sketch the emission spectra for an idealized blackbody at two different temperatures where $T_2 > T_1$. **(3 marks)**

ii. Describe in your own words how these two spectra differ. **(2 marks)**

e) Figure 1 shows the pattern of lines in the Balmer series of the atomic hydrogen spectrum

i. State the lines in the Balmer series with the lowest energy of light. Explain your answer. **(1 marks)**

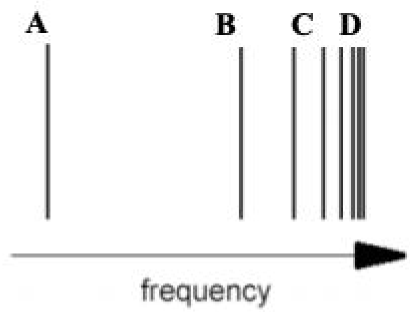


Figure 1

ii. The emission spectrum of atomic hydrogen is divided into several spectral series, with wavelengths given by the Rydberg formula, Determine the wavelength of light absorbed in an electron transition from n=3 to n=6 in a hydrogen atom **(3 marks)**

f) Use the Bohr model result, $E_n = -\frac{13.6}{n^2}\,eV$ to derive Balmer's formula $\lambda = 364.56\left(\frac{n^2}{n^2-4}\right)nm.$ **(3 marks)**

g) Obtain the expression of the maximum energy of a photon emitted by the hydrogen atom in eV from any state n. **(3 marks)**

h) A beam of particles is incident normally on a thin metal foil of thickness t. If N_0 is the number of nuclei per unit volume of the foil, show that the fraction of incident particles scattered in the direction (θ,ϕ) is $\sigma\,(\theta,\phi)N_0 t d\Omega$, where $d\Omega$ is the small solid angle in the direction (θ,ϕ). **(4 marks)**

i) A hydrogen atom in the p state is placed in a cavity. Find the temperature of the cavity at which the transition probabilities for stimulated and spontaneous emissions are equal. **(4 marks)**

QUESTION TWO (20 MARKS)

a. Show that in the usual stationary state perturbation theory, if the Hamiltonian can be written $H = H_0 + H'$ with $H_0\phi_0 = E_0\phi_0$, then the correction $\Delta\phi_0$ is, $\Delta E_0 \approx \langle\phi_0|H'|\phi_0\rangle$**(6 marks)**

b. For a spherical nucleus, the nucleous may be assumed to be in a spherical potential well of radius R given by $V_{sp} = \begin{cases} 0, & r < R \\ \infty, & r > R \end{cases}$

For a slightly deformed nucleus, it may be correspondingly assumed that the nucleons are in an elliptical well, again with infinite wall height, that is,

$$V_{el} = \begin{cases} 0, & \text{inside the ellipsoid } \frac{x^2+y^2}{b^2} + \frac{z^2}{a^2} = 1 \\ \infty, & \text{otherwise} \end{cases}$$

, where $a \cong R\left(1+\frac{2\beta}{3}\right), b \cong R\left(1-\frac{\beta}{3}\right), and\ \beta \ll 1.$

Calculate the approximate change in the ground state energy E_0 due to the ellipticity of the non-spherical nucleus by finding an appropriate H' and using the result obtained in (a). HINT: try to find a transformation of variables that will make the well look spherical.

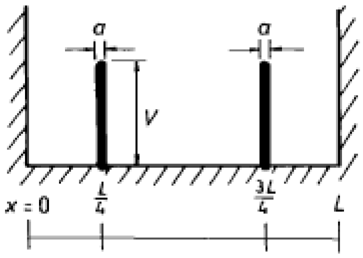
(14 Marks)

QUESTION THREE (20 MARKS)

a. An infinitely deep one-dimensional square has walls at $x = 0$ and $x = L$. Show that the energy levels and wave functions for a one-dimensional infinite potential well are respectively, $E_n^{(0)} = \frac{\pi^2 \hbar^2}{2mL^2} n^2$, and $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$, $n = 1,2, \ldots$ **(8 marks)**

b. For the potential wall in (a) solve for the energy E, using the WKB method and compare it with the exact solution **(5 marks)**

c. Two small perturbing potentials of width a and height V are located at $x = L/4$, $x = \frac{3}{4}L$, where a is small $(a \ll \frac{L}{100}, say)$ as shown in the Figure. Using perturbation methods, estimate the difference in the energy shifts between the $n = 2$ and $n = 4$ energy levels due to this perturbation. **(7 marks)**



QUESTION FOUR (20 MARKS)

a. For a particle of mass m moving in the potential, $V(x) = \begin{cases} kx, & x > 0 \\ \infty, & x < 0 \end{cases}$, where k is a constant. Optimize the trial wavefunction $\phi = x \exp(-ax)$, where a is the variable parameter, and estimate the ground state energy of the system. **(10 marks)**

b. Using the WKB method, calculate the transmission coefficient for the potential barrier

$$V(x) = \begin{cases} V_0 \left(1 - \frac{|x|}{\lambda}\right) & |x| < \lambda \\ 0, & |x| < \lambda \end{cases}$$ **(10 marks)**

QUESTION FIVE (20 MARKS)

a. Write the general procedure to follow to calculate the differential cross section in born approximation and method of partial waves? **(5 marks)**

b. Use the Born approximation to calculate the differential and total cross section for scattering in a potential $V(r) = \alpha/r^2$ where α a constant is. Given $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. **(7 marks)**

c. In the Born approximation, calculate the scattering amplitude for scattering from the square well potential $V(r) = \begin{matrix} -V_0 & 0 < r < r_0 \\ 0 & r > r_0 \end{matrix}$ **(8 marks)**