CHUKA UNIVERSITY

UNIVERSITY EXAMINATION
RESIT/SUPPLEMENTARY / SPECIAL EXAMINATIONS
EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION ARTS, BACHELOR OF ARTS (MATHEMATICS ECONOMICS)

MATH 220/222: VECTOR ANALYSIS

STREAMS: BSC MATHS, B.ED SC/ARTS, BA ECOM MATH Y2S2

TIME: 2 HOURS

DAY/DATE: TUESDAY 10/08/2021 **11.30 A.M - 1.30 P.M.**

INSTRUCTIONS

Answer ALL questions

Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Distinguish the following terms as used in vector analysis:
- (i) Linearly dependent vectors and Linearly independence vectors (2 marks)
- (ii) The dot product and cross product of two vectors $\overrightarrow{A}$ and $\vec{B}$ (2 marks)
- (iii) The gradient of a scalar function $\emptyset$ and the divergence of the vetor$\overrightarrow{V}$ (2 marks)
- (iv) An irrotational vector and a solenoidal vector $\overrightarrow{V}$ (2 marks)

(b) (i) Show that addition of two vectors is commutative (4 marks)

(ii) Prove that if $\vec{a}$ and $\vec{b}$ are non-collinear vectors then $x\vec{a}+ y\vec{b}=0\ implies\ x=y=0$ (4 marks)

(c) Given $\vec{A}=A_1\hat{\imath}+A_2\hat{\jmath}+A_3\hat{k}$ and $\vec{B}=B_1\hat{\imath}+B_2\hat{\jmath}+B_3\hat{k}$, show that

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\end{vmatrix}$$ (4 marks)

(d) Using vector method find the area of the triangle having vertices $P(2,3,5), Q(4,2,-1), R(3,6,4)$ (5 marks)

(e) If $\vec{A}=(2x^2y-x^4)\hat{\imath}+(e^{xy}-ysinx)\hat{\jmath}+(x^2cosy)\hat{k}$, find $\frac{\delta^2 A}{\delta x\delta y}$ (3 marks)

(f) State without proof the Green's theorem in a plane (2 marks)

QUESTION TWO: (20 MARKS)

(a) Find the equation for the plane perpendicular to the vector $\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ and passing through the terminal point of the vector $B = \hat{\imath} + 5\hat{\jmath} + 3\hat{k}$ (5 marks)

(b) Find the work done in moving a particle in a force field given by

$$\vec{F} = (2xy)\hat{\imath} - (5z)\hat{\jmath} + (10x)\hat{k} \text{ along } x = t^2 + 1, y = 2t^2, z = t^3 \text{ from } t = 1 \text{ to } t = 2$$

(5 marks)

(c) Show that $\nabla \cdot \nabla\emptyset = \nabla^2\emptyset$, where $\nabla^2 = \frac{\delta^2}{\delta x^2} + \frac{\delta^2}{\delta y^2} + \frac{\delta^2}{\delta z^2}$ denotes the laplacian operator. (5 marks)

(d) Given that $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$. Evaluate $\int_C \vec{A} \cdot d\vec{r}$ between the points (0,0,0) and (1,1,0) (5 marks)

QUESTION THREE: (20 MARKS)

(a) (i)Given that $\vec{A} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ .Find the constants *a, b and c* such that the vector $\vec{A}$ is irrotational. (3 marks)

(ii) Hence show that the vector $\vec{A}$ in (c,(i)) can be expressed in as a gradient of a scalar function $\emptyset$ (7 marks)

(b) State without proof the Frenet-Serret formulas (3 marks)

Hence given the space curve defined by $x = 3cost, y = 3sint, z = 4t$. Find

(i) The tangent vector $\vec{T}$ (3 marks)

(ii) The principal normal $\vec{N}$ (4 marks)

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