CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF BACHELOR OF SCIENCE DEGREE IN MATHEMATICS

MATH 205: ELEMENTS OF SET THEORY

STREAMS: "AS ABOVE"

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 31/3/2021 **8.30 AM – 10.30 AM**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) For each of the following cases, determine whether it represents a function. If it is a function, state whether it is injective

i. To each of the 190 student in math 205, assign a number corresponding to his/her age

ii. To each student in Chuka university, assign a registration number

iii. To each student in first year, assign the semester course units

iv. To each book written by a single author, assign the author

v. To each positive number, assign its square root (5 marks)

b) Given the sets $A_n = \{n+1, n+2, ...\}$, where n is a positive integer evaluate

i. $\bigcup_{n=3}^{10} A_n$ and $\bigcap_{n=1}^{10} A_n$

ii. $\bigcup_n A_n$ and $\bigcap_n A_n$ (4 marks)

c) Find the domain of the function $f: R \to R$ defined by $f(x) = \frac{4}{\sqrt{x^2 - 4}}$ (2 marks)

d) Consider the set $A = \left\{11 + (-1)^n \frac{1}{n}\right\}$ where n is a positive integer

   i. Find the supremum and the infimum of A (2 marks)

   ii. Find all the limit points of A (1 marks)

e) With an appropriate example, show that a bounded sequence is not necessarily convergent (2 marks)

f) Consider the function $f: R \to R$ defined by $f(x) = x^2$

   i. Find the largest set D such that $f: D \to R$ is injective

   ii. Find the smallest set T such that $f: R \to T$ is onto (3 marks)

g) Prove that the set of integers is countable (3marks)

h) Prove that if the limit of a sequence exists, then it is unique (3marks)

i) State the Axiom of choice (1 marks)

QUESTION TWO (20 MARKS)

a) Distinguish the following

   i. Injective and surjective functions (2 marks)

   ii. A countable and uncountable set (2 marks)

   iii. A linearly ordered set and a poset (2 marks)

b) Let $f: A \to B$ and $g: B \to C$ be two invertible functions with inverses f^{-1} and g^{-1} respectively.

   i. Prove that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$ (4 marks)

   ii. Given that $f(x) = 2x - 3$ and $g(x) = \frac{2x-7}{5x+7}$ find $(f \circ g)^{-1}$ (4 marks)

c) Let A and B be sets in a universal set U, prove that $\chi_{A \cap B} = \chi_A \chi_B$ where χ_A is the characteristic function of A and $\chi_A \chi_B$ is the product of functions (6 marks)

QUESTION THREE (20 MARKS)

a) Prove that a composition of two injective functions is injective (4 marks)

b) Prove the distributive laws i.e.

   i. $B \cap (\bigcup_k A_k) = \bigcup_k (B \cap A_k)$ (5 marks)

   ii. $(\bigcap_k A_k)^c = \bigcup_k (A_k)^c$ (5 marks)

c) Let $A_m = \{m, 2m, 3m, \ldots\ldots\ldots\ldots : m \in N\}$, determine and explain the following sets

i. $A_3 \cap A_7$ (2 marks)

ii. $A_3 \cup A_7$ (2 marks)

iii. $\bigcup_m A_m$ (1 marks)

iv. $\bigcap_m A_m$ (1 marks)

QUESTION FOUR (20 MARKS)

a) The prerequisites in a college is a familiar partial ordering of available classes. Let M be a set of mathematics courses at XYZ college . Define $A \prec B$ if class A is a prerequisite of class B, below is a list of mathematics courses and their prerequisites

Class	Prerequisite
Math 122	None
Math 201	Math 122
Math 205	Math 122
Math 206	Math 205
Math 301	Math 201
Math 302	Math 301
Math 401	Math 201,Math 205
Math 403	math 206, Math401

Required:

i. Draw an Hasse diagram for the partial ordering of these classes (2 marks)

ii. Find all the minimal and maximal elements of these classes (2 marks)

iii. Determine the first and last element if they exist. (2 marks)

b) Let A be a non empty set and P(A) be the power set of A ordered with set inclusion.

i. Find the first element and the last element of P(A) explaining each case

ii. Is A well ordered?Verify

iii. Partially ordered? Verify (4 marks)

c) Let λ be an ordinal number. Prove that $\lambda + 1$ is the immediate successor of λ (3 marks)

d) Given a sequence intervals $I_1, I_2, \ldots\ldots\ldots$ such that $I_1 \supseteq I_2 \supseteq \ldots\ldots\ldots$, it is called a 'nested' sequence.

i. Give an example of a nested sequenced of nonempty open intervals whose intersection is empty. (2 marks)

ii. Give an example of a nested sequenced of open intervals whose intersection is not empty (2 marks)

iii. Prove that intersection that a nested sequence of closed intervals of the form $I_1 \supseteq I_2 \supseteq \ldots\ldots\ldots$ is not empty (3 marks)

QUESTION FIVE (20 MARKS)

a) Prove that the intervals [0,1]and(0,1] are equivalent. (5 marks)
b) Prove that The unit interval [0,1] is non denumerable (5 marks)
c) Prove that a countable union of finite sets is countable (5 marks)
d) Prove that every infinite set contains a countable set (5 marks)

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