MAIN EXAM

INTRODUCTION TO ANALYSIS

QUESTION ONE (30 MARKS)

a. Differentiate between open sentence and a universal (2marks)
b. What are the requirements of a set (3 marks)
c. The sets L ,M and N in a universal set consisting of the first 10 lower case letters of the alphabet are L={a,b,c}, M= {b,c,q,z)
   N = {a,d,e,f}
   Determine the numbers of the following sets.
   i. MÚN
   ii. LÚN
   iii. Ŀ
   iv. LnMnN˙
   v. (LÚMÚN)˙
   vi. MnN

(6 Marks)

d. A salesman daily wages is composed of a fixed amount and a variable component, which is dependent on the number of office cream units sold. He finds out that when he sells 10 units on a given day, he earns Kshs. 600 whereas when he doubles his sales, his earnings increase only by Kshs. 100.
   Determine
   i. Fixed daily earnings
   ii. Level of commission per unit sold hence.
   iii. What are the salesman's earnings if he sells 30 units.
   iv. On a given day, the salesman is determined to earn Kshs. 3,500. Suppose on the previous day he had a guaranteed order of 20 units, how many more must he sell in order to achieve his target earnings (6 marks).

e. List and explain six assumptions of C-V-P – cost volume analysis (3 marks).

f. Prove that for n€N
   $1+4+9+ \ldots . +n^2 = {}^1/_6\,(2n^3 + 3n^2+n)$ (4marks)

g. Let x,y and Z € F
   Proof that
   i. If xǂ0 and xy = xz then y = z

ii. If $x \neq 0$and $xy = x$ then $y = 1$
iii. If $x \neq 0$ and $xy=1$ then $y = {}^1/_x$
iv. If $x \neq 0$ then ${}^{1/1}/_x = x$ (6 marks)

QUESTION TWO (20 MARKS)

a. Define the completeness of an axiom (2 marks)
b. List four implications of Archimedean property a real numbers (4 marks)
c. State the Bolzano –Weierstrass theorem (2 marks)
d. Show that $\lim_{n \to \infty} = n\sqrt{n} = 1$ (4 marks)

e. Let P and Q be propositions construct the truth table for the proposition $(P \wedge Q) = ((P \wedge Q)$ (4 marks)

f. Show that if 3n is an odd integer then n is an odd integer. (4 marks)

QUESTION THREE (20 MARKS)

a. Prove that "7 is a divisor of $3^{2n}-2^n$ (3 marks)
b. Let A, B and C be subjects of a universal set U. gone up with the below rules
   i. Commutative law
   ii. Associative law
   iii. Idempotent law
   iv. Demorgan law
   v. Distributive law (5 marks)
c. A survey was conducted on the newspaper readership of 3 dailies, the mirror, the citizen and the times M, C, T respectively and the following data were obtained.
The number of people who read M, C and T are 55, 45 and 39 respectively.
The number that read M and T = 19
The number that read C and M =15
The number that read C and T = 14
Those who read all the 3 dailies were found to be 4 people only.

Required:

Determine the number of people who

i. Read the Mirror only
ii. Read Citizen or times but not the Mirror
iii. Total number of people interviewed 45 people read none of the paper. (8 marks).

d. Differentiate between a converging and diverging sequence (4 marks)

QUESTION FOUR (20 MARKS)

a. Define what is a topological space and give three conditions of topology to be open (4 marks)

b. Show that Lim $(1-{}^{1}/_{2}{}^{n}) = 1$
Using the Archimedean property (6 marks)

c. Discuss the convergence and divergence of
   i. Harmonic series (3 marks)
   ii. P series with P= 2 (3 marks)

QUESTION FIVE (20 MARKS)

a. Differentiate between a multivariate function and a logarithmic function (4 marks)

b. What are the applications of linear functions in the business world and explain how linear functions are applied (6 marks).

c. Show that $\lim_{n \to \infty} \frac{\cos {}^{nx}/_{2}}{n^2} = 0$
(4 marks)

d. Differentiate between least upper bound (supremum) and greatest lower bound (infimum) (6 marks)