CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION SCIENCE/ARTS, BACHELOR OF SCIENCE CHEMISTRY/INDUSTRIAL CHEMISTRY, BACHELOR OF ARTS (MATHS-ECONS) AND BACHELOR OF SCIENCE (ECON STATS)

MATH 201: LINEAR ALGEBRA I

STREAMS: AS ABOVE Y2S2 **TIME: 2 HOURS**

DAY/DATE: MONDAY 05/07/2021 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) Consider the system in unknowns $x \text{ and } y$

$$ax + 9y = b$$
$$2x + ay = 4$$

Find which values of (a, b) that give the three types of solutions to the system (4 marks)

b) Evaluate the WROSKIAN $W(\sin x, \sin 2x, \cos x, \frac{\pi}{4})$ (4 marks)

c) Distinguish the Kernel and Range of a transformation T. Hence prove that if $T: U \to V$ is a linear transformation, then the kernel of T is a subspace of U. (5 marks)

d) Show that the subset $W = \{(x, y) : x + y = 0, x, y \in R\}$ is a subspace of R^2 (3 marks)

e) Determine if $T: R^3 \to R^2$ defined as $T(x_1, x_2, x_3) = (x_1 + x_3,\ 2x_2, -x_3)$ is a linear transformation. (4 marks)

f) Verify whether or not the vector (1,2,-1) is a linear combination of the vectors {(1,1,-1),(2,2,1),(-1,-1,2)}. Can these vectors form a basis for $\mathbb{R}^3$ (5 marks)

g) Given the following basis for $\mathbb{R}^3$ $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ and $B' = \{(1,0,1,),(2,1,2),(1,2,2)\}$ find a transition matrix from B to B' (5marks)

QUESTION TWO: (20 MARKS

a) For which values of a does the below system has

(i) No solution

(ii) Unique solution

(iii) Infinitely many solutions.

$$ax_1 + x_2 + x_3 = -1$$

$$x_1 + ax_2 + x_3 = 4$$

$$x_1 + x_2 + ax_3 = 1$$

(8 marks)

b) (i)Using of row reduction method, find the inverse for the matrix A = $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 6 \\ 1 & 1 & 4 \end{bmatrix}$

(ii) Hence or otherwise find all the solutions to the system $A^2x = b$ where b is the vector (0,1,1) (8 marks)

(c) For a matrix $A_{(m \times n)}$, prove that

i) If A is invertible, then A$\underline{x}$ = $\underline{b}$ has a unique solution for any b (2 marks)

ii) If A is row equivalent to an identity matrix I_n, then A is invertible (2 marks)

QUESTION THREE: (20 MARKS)

a) Find the inverse of the following matrix by first getting the adjoint

$$\begin{pmatrix} 2 & 1 & -2 \\ 3 & 2 & 2 \\ 5 & 4 & 3 \end{pmatrix}$$

Hence or otherwise, solve the following system of linear equations

$$2x_1 + x_2 - 2x_3 = 10$$

$$3x_1 + 2x_2 + 2x_3 = 1$$

$$5x_1 + 4x_2 + 3x_3 = 4$$

(7 marks)

b) Let $F: \mathbb{R}^5 \to \mathbb{R}^4$ be a linear mapping defined by $F(x,y,z,w,t) = (x+2y+w-t, 2x-y+3z+t, -x-2z+t, 2w+8t)$. Find

i. The basis and dimension of the kernel of F

ii. A basis and dimension of the image of F

iii. Using the parts i) and ii) above, verify the dimension theorem (7 marks)

c) Show that the vectors $\{1, x-1, x^2-1\}$ is a basis for the vector space $P^2(x) =$ the space of all polynomials in x of degree less or equal to 2 (6 marks)

QUESTION FOUR: (20 MARKS)

a) Solve the following system of equations of the planes by use of Gauss Jordan elimination method

$$2x_1 + x_2 + x_3 = 1$$

$$-x_1 + 2x_2 - 3x_3 = 3$$

$$x_1 + 3x_2 - 2x_3 = 4$$

Hence give the geometrical interpretation of the solution of the planes (5 marks)

b) (i)Prove that if S is a subset of a vector space V, then the set spanned by S is a subspace of V. (3 marks)

ii) Let V = R^3 and S = {(2, 3, 5), (1, 2, 4),(-2, 2, 3)}. Determine if (10, 1, 4) $\in span$ S (4 marks)

c) Let $T: R^2 \to R^2$ be a linear transformation defined by $T(x,y) = (x-2y, -x+3y)$

i. Find the matrix of T relative to the basis $B = \{(1,0), (1,1)\}$ (3 marks)

ii. Find the matrix of T relative to the basis $B' = \{(1,-1), (1,2)\}$ (3 marks)

iii. Find the transition matrix P from the basis B to the basis B' and verify the relation $P^{-1}[T]_B P = [T]_{B'}$ (3 marks)

QUESTION FIVE: (20 MARKS)

a) Use Cramer's method to solve the system of equation

x + y -2z = -3

w + 2x - y = 2

2w+ 4x +y -3z = -2

w - 2x – 7y - z = 5 (8 marks)

b) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$

$$x_1 + x_2 - 2x_3 - x_5 = 0$$

$$x_3 + x_4 + x_5 = 0$$

(7 marks)

c) Prove that any two bases defined on the same vector space have the same number of vectors.

(5 marks)
