CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 201/210: LINEAR ALGEBRA I

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 11/08/2021 **8.30 A.M – 10.30 A.M.**

<u>INSTRUCTIONS</u>
Answer ALL the questions

QUESTION ONE: (30 MARKS)

a) Consider the system in unknowns x and y

$$x + ay = 4$$
$$ax + 9y = b$$

Find which values of a does the system have a unique solution, and for which pairs of values (a, b) does the system have more than one solution. (5 marks)

b) Evaluate the WROSKIAN $W(e^{x}, e^{-x}, e^{-2x}, 0)$ (5 marks)

c) Show that the subset $W = \{(x, y) : x \geq 0, y \geq 0, x, y \in R^2\}$ is not a subspace of R^2 (5 marks)

d) For any vector $\boldsymbol{v} = (v_1, v_2)$ in R^2, define $T: R^2 \rightarrow R^3$ defined by

$T(v_1, v_2) = (v_1 - v_2, 3v_1 - 2v_2, v_1 + 2v_2)$, show that is a linear transformation (5 marks)

e) Determine if $p_1 = 1 - t, p_2 = 2 - t + t^2$ and $p_3 = 2t + 3t^2$ is a basis for the vector space $P_2(t)$ of polynomials of degree less or equal to 2 (5 marks)

QUESTION TWO: (20 MARKS)

(a) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Is the equation $A\boldsymbol{x} = \boldsymbol{b}$ consistent for all values of b_1, b_2, b_3? Verify (6 marks)

(b) By use of the concept of rank of matrix, determine the type of solution to the following system of equations

$$2x_1 + x_2 + x_3 = 1$$

$$-x_1 + 2x_2 - 3x_3 = 3$$

$$x_1 + 3x_2 - 2x_3 = 4$$

(7 marks)

c) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$

$$x_1 + x_2 - 2x_3 - x_5 = 0$$

$$x_3 + x_4 + x_5 = 0$$

(7 marks)

QUESTION THREE: (20 MARKS)

a) Using of row reduction method, find the inverse for the matrix A = $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -3 \\ 2 & 1 & 5 \end{bmatrix}$ hence solve the system

$$x + y + 2z = 2$$
$$x + y - 3z = 2$$
$$2x + y + 5z = 5$$

(10 marks)

b) Let $F: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear mapping defined by $F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$. Find

i. The basis and dimension of the kernel of F

ii. A basis and dimension of the image of F

iii. Using the parts i) and ii) above, verify the dimension theorem (10 marks)
