CHUKA **UNIVERSITY**

SUPPLEMENTARY / SPECIAL EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF BACHELOR DEGREE IN

MATH 125: DISRETE MATHEMATICS

STREAMS:

TIME: 2 HOURS

DAY/DATE: MONDAY 16/11/2020 **2.30 P.M - 4.30 P.M.**

INSTRUCTIONS:

QUESTION ONE (30 MARKS

a) Determine the validity of the following argument

S_1 : Some crazy people are dangerous

S_2 : All fanatics are crazy

Conclusion: some fanatics are not dangerous. (3 marks)

b) Prove the proposition that the sum of n positive even integers is n(n+1) (4 marks)

c) Find all the integers such that $2 < 8 - 3n \leq 18$ (3 marks)

d) Give an example of a non-trivial relation on the set $A = \{1,2,3\}$ which is

i. Both symmetric and antisymmetric (2 marks)

ii. Neither symmetric nor antisymmetric (2 marks)

e) Solve the linear congruence equation $4x \equiv 6(\text{mod } 10)$ (4 marks)

f) Find the product of the polynomials $f(x) = 4x^3 - 2x^2 + 3x - 1$ and $g(x) = 3x^2 - x - 4$ over Z_5 (4 marks)

g) Prove that $(a+b)' = a' * b'$ (5 marks)

h) Given a binary on the set of integers given by $a * b = a + b - ab.$ show that * is commutative and associative (3 marks)

QUESTION TWO (20 MARKS)

a) Let $A = \{1,2,3\}$ $B = \{a, b, c, d\}$ and $C = \{x, y, z, w\}$. Suppose R and S are relations from A to B and from B to C respectively defined by $R = \{(1, a), (2, a), (2, c), (2, d), (3, b)\}$ and $S = \{(a, x), (a, z), (c, w), (d, y)\}$.

i. Draw an arrow diagram to represent the relation $R \circ S$ (2 marks)

ii. Show that the product of the matrix representation of R and S has the same representation as the matrix $R \circ S$ (4 marks)

iii. Find the domain and range of $R \circ S$ (2 marks)

b) Let $S = \{1,2,...,9\}$ and R be a relation on S defined by $(a,b) \approx (c,d)$ if and only if $a + d = b + c$.

i. Show that $\approx$ is an equivalence relation (6 marks)

ii. Find the equivalence class of [2,5] (2 marks)

c) Use Venn diagrams to determine the validity of the following arguments

S_1: Some innocent people go to Jail

S_2: Mary is innocent

S_3:All people in jail are bad people

Conclusion: Mary is not a bad person. (4 marks)

QUESTION THREE(20 MARKS)

a) Use mathematical induction to prove that 43 divide $6^{n+1} + 7^{2n+1}$ (6 marks)

b) Let a=195 and b=968. Use the division algorithm to find the gcd(a,b) and therefore find integers m and n such that d=am +bn (6 marks)

c) Consider the third order homogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$

i. Find the general solution (4 marks)

ii. Find the initial solution given $a_0 = 3, a_1 = 4, a_2 = 12$ (4 marks)

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