CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF BACHELOR OF EDUCATION (SCIENCE &ARTS), BACHELOR OF ARTS (ECON/MATH), BACHELOR OF SCIENCE (MATHEMATICS) AND BACHELOR OF ARTS (ECONOMICS & SOCIOLOGY)

MATH 123: VECTORS AND MECHANICS

STREAMS:

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 31/3/2021 **11.30 AM – 1.30 PM**

INSTRUCTIONS:

- Answer Question **ONE** (Compulsory) and any other **TWO** Questions
- Take g =10 NKg^{-1}

QUESTION ONE (30 MARKS) COMPULSORY

(a) Distinguish between vectors and scalars giving an example of each. [4 Marks]

(b) Define the term mechanics. [2 Marks]

(c) Determine the angle between the vectors

$\vec{A}$= 3i-2j-+ 4k and $\vec{B}$= si + 7j – 3k. [3 Marks]

(d). A force whose point of application is (-1, 2, 3) is given by $\vec{F}$= 4$\tilde{\imath}$+ 2$\tilde{\jmath}$-7k. Determine the magnitude of the moment of the force about the point (3, -2, 1). [4 Marks]

(e). Given $\vec{A}, \vec{B}$ and $\vec{C}$ are the three sides of a triangle and θ the angle between vectors $\overrightarrow{A}$ and $\vec{B}$, prove that $C^2 = A^2 + B^2 - 2AB\,Cos\theta$. [3 Marks]

(f) A particle moves along a curve whose parametric equations are $x = e^{-t}, y = 2\cos 3t$ *and* z=2 sin 3t, where t is the time. Find the magnitudes of its velocity and acceleration at t= 0.

[5 Marks]

(g) Given ϕ (x, y, z) = $3x^2y - y3z^2$, find $\nabla\phi$or grad $\phi$the point (1, -2, -1). [3 Marks]

(h) A 100kg mass is suspended from the centre of a rope as shown in fig 1.

Determine the tension T in the rope. [3 Marks]

(i) if $\vec{A}$= x^2 zi −-$2y^3$ z^2 j+ xy^2zk, Find div $\vec{A}$ at the point (1,-1, 1). [3 Marks]

QUESTION TWO (20 MARKS)

(a) Show that the following lines intersect and if they do determine the point of intersection

L_1: $\tilde{r} = 2\tilde{\imath} - 3\tilde{\jmath} + 4\tilde{k} + \lambda$ (6i + 7j -k)

L_2: $\tilde{r}$= $2\tilde{\imath}$-$12\tilde{\jmath}$ -$\tilde{k}$ + μ(-$3\tilde{\imath}$+ $\tilde{\jmath}$+$3\tilde{k}$) [8 Marks]

(b) A stone is dropped from the top of a tower 125m. high. When it has fallen 20m, a second stone is thrown vertically downwards with an initial velocity Vm/s from the top of the tower. If the two stones reach the ground at the same time, calculate the velocity with which the second stone hits the ground. [6 Marks]

(c) (i) Define the term couple giving an example. [3 Marks]

(ii) Show that a force of magnitude 36N acting at the vertices of the square ABCD of length 20cm (see fig2) form a couple and find the magnitude of the couple.

[3 Marks]

QUESTION THREE (20 MARKS)

(a) If $\vec{A}$= x^2yi - 2x zj + 2yz k, find

Curl curl $\vec{A}$. [4 Marks]

(b) Find the perpendicular distance of the point A(4, -3, 10) from the line whose vector equation is $\tilde{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$ [7 Marks]

(c). A particle moves along the curve x = 2sin 3t, y =2 cos 3t, z =8t at any time t>0. Determine

(i) the magnitude of its velocity. [3 Marks]

(ii) the magnitude of its acceleration. [3 Marks]

(d) Given $\overrightarrow{A}, \vec{B}$ and $\vec{C}$ are the three sides of a triangle and θ is the angle between vectors $\vec{A}$ and $\vec{B}$, show that

$$\frac{A}{Sin\ A} = \frac{B}{Sin\ B} = \frac{C}{Sin\ C}$$ [3Marks]

Question Four (20 Marks)

(a) Prove the associative property of vector addition. [3 Marks]

(b) Determine the area of the triangle whose vertices are A (2, -1, 3), B(5, -2, 7) and C(1, 5, -3) [4 Marks]

(c) (i) State Lamis theorem. [2 Marks]

(iii) A body of mass 4kg hand from a string attached to a point on a vertical wall. The string will break when its tension exceeds 50N. If the body is pulled away from the wall by a horizontal force FN, what is the value of F at the point when the string breaks? [4 Marks]

(d) A ladder of negligible weight stands on a rough ground with its upper end against a smooth wall. The foot of the ladder is 1m away from the wall and the top is 3m above the ground. A boy whose mass is 50kg climbs two-third of the way up the ladder, at which point the ladder begins to slide down. Find:

(i) the reaction at the wall. [3 Marks]

(ii) the reaction at the ground [2 Marks]

(iii) the coefficient of friction when sliding begins. [2 Marks]

Question Five (20 Marks)

(a) The resultant of two forces F_1 and F_2 acting at a point is R. Find an expression for:

(i) the angle $\propto$ between F_1 and F_2. [3 Marks]

(ii) the direction of the resultant to the horizontal. [3 Marks]

(iii) the line of action of the resultant force. [2 Marks]

(b) (i) When t=0, a particle A moves from point O along a straight line with an initial velocity umls and constant acceleration amls^2. When t=4, a particle B moves from O along the same straight line with an initial velocity $\frac{1}{2}$ umls and a constant acceleration 20 mls^2. Given that when t= 16, A is ahead of B, obtain in terms of u and a an expression for the distance between the particles at that time. [3 Marks]

(II) Given also that this distance is 12m, and that the velocity of A when t=16 is 10mls, calculate:

(i) the value of u and of a. [3 Marks]

(ii) the distance between the particles when t =18. [3 Marks]

(iii) the difference between their velocities when t=18. [3 Marks]

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