CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF MASTER OF SCIENCE (PHYSICS)

PHYS 811: MATHEMATICAL PHYSICS

STREAMS: MSC (PHYS) TIME: 3 HOURS

DAY/DATE: TUESDAY 06/04/2021 8.30 A.M. – 11.30 A.M.

INSTRUCTIONS: Answer question ONE (Compulsory) and any other THREE questions

QUESTION ONE (15 MARKS)

a) Use Cauchy-Riemann conditions to show that $f(z)=z^2$ is analytic in the entire z-plane

[3 marks]

b) Show that the following four matrices form a group under matrix multiplication [4 marks]

$$E=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}, B=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}, C=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

c) Prove the following recurrence relation for Bessel function

$$J_n'(x)=\frac{-n}{x}J_n(x)+J_{n-1}(x)$$

Where the prime denotes the differentiation with respect to x [4 marks]

Given: $J_n(x)=\sum_{r=0}^{\infty}(-1)^r\left(\frac{x}{2}\right)^{n+2r}\frac{1}{r!\sqrt{(n+r+1)}}$

e) Find $\frac{\frac{dy}{dx} \wedge d^2 y}{dx^2}$ for $y = e^{-x^2}$ at the point x=0.05 from the data of the table given below

[4 marks]

X	$y=e^{-x^2}$	Δ	Δ^2	Δ^3	Δ^4
0	1.00000				
0.05	0.99750	-250			
0.10	0.99005	-745	-495		
0.15	0.97775	-1230	-485	+10	
0.20	0.96079	-1696	-466	+19	+9
0.25	0.93941	-2138	-442	+24	+5
0.30	0.91393	-2548	-410	+32	+8

QUESTION TWO (15 MARKS)

a) State and prove the residue theorem [5 marks]

b) Evaluate the integral

$$\int_0^\infty \frac{\sin x}{x} dx$$

using the residual theorem [10 marks]

QUESTION THREE (15 MARKS)

a) Construct the Green's function for the problem stated mathematically as

$$\frac{d^2 y}{dx^2} - k^2 y = f(x)$$

where f(x) is a known function and y satisfies the boundary conditions $y = (\pm\infty)$ [7 marks]

b) Define the shifting property of the Laplace transform and use it to find the Laplace transform of $e^{-x}\cos x$ [4 marks]

c) Obtain Rodrigues' formula for the Legendre polynomials [4 marks]

QUESTION FOUR (15 MARKS)

a) Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{\cos x}{\left(x^2+a^2\right)\left(x^2+b^2\right)} dx$$

Using the residual theorem [7 marks]

b) Define isomorphism and show that the group $(i,-1,-i,1)$ is isomorphic to the cyclic group $(A, A^2, A^3, A^4=E)$ [3 marks]

c) Using the table given below, evaluate the integral

$$\int_0^{1.0} \frac{x^3}{e^x-1} dx$$

By using Simpson's one- third rule [5 marks]

X	$f(x)=\frac{x^3}{e^x-1}dx$
0	0
0.25	0.055013
0.50	0.192687
0.75	0.377686
1.00	0.581977

QUESTION FIVE (15 MARKS)

A sphere of radius *a* is centred at O. It is cut into two equal halves by the x-y plane. The upper part is maintained at potential $+V_o$ and the lower part at potential $-V_o$. Calculate the potential at a point inside the sphere in the following steps:

i) Write the Laplace's equation satisfied by the potential in spherical polar coordinates and make use of the method of separation of variables to separate it into the $\varphi-,\theta-,\wedge r-¿$ equations. [4 marks]

ii) Solve the $\varphi-,\theta-,\wedge r-¿$ equations. [5 marks]

iii) Make use of the boundary conditions to find the potential. [6 marks]
