CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF MASTERS OF SCIENCE IN PHYSICS

PHYS 832: QUANTUM MECHANICS

STREAMS: MSc (PHYSICS) TIME: 3 HOURS

DAY/DATE: THURSDAY 08/04/2021 8.30 A.M. – 11.30 A.M.

INSTRUCTIONS:

- **Answer Any Four Questions**
- **Do not write anything on the question paper**
- **This is a closed book exam, No reference materials are allowed in the examination room**
- **There will be No use of mobile phones or any other unauthorized materials**
- **Write your answers legibly and use your time wisely**

QUESTION ONE (15 MARKS)

a) A particle is in the nth energy state $\psi n(x)$ of an infinite square well potential with width L. Determine the probability $P_n(1/a)$ that the particle is confined to the first 1/a of the width of the well. Comment on the n-dependence of P(1/a) [6 marks]

b) A particle has the wave function, $\psi(r) = N e^{-\alpha r}$ where N is a normalization factor and α is a known real parameter.

(i) Calculate the factor *N*. [2 marks]

(ii) Calculate the expectation values, $\langle x \rangle, \langle r \rangle, \langle r^2 \rangle$ in this state. [3 marks]

(iii) Calculate the uncertainties $(\Delta x)^2$ and $(\Delta r)^2$. [2 marks]

(iv) Calculate the probability of finding the particle in the region, $r > \Delta r$ [2 marks]

QUESTION TWO (15 MARKS)

Starting with the canonical commutation relations for position and momentum, work out the following commutators

i. $[L_z, x] = i\hbar y$, $[L_z, y] = -i\hbar x$, $[L_z, z] = 0$

$[L_z, p_x] = i\hbar\, p_y, [L_z, p_y] = -i\hbar\, p_x, [L_z, p_z] = 0$ [3 marks]

ii. Use these results to obtain $[L_z, L_x] = i\hbar L_y$,directly from equations

$L_x = y\, p_z - z\, p_y, L_y = z\, p_x - x\, p_z \wedge L_z = x\, p_y - y\, p_x$ [3 marks]

iii. Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$ (where of course $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$) [4 marks]

iv. Show that the Hamiltonian $H = (p^2/2m) + V$ commutes with all three components of L, provided that V depends only on r. (Thus H, L^2 and L_z are mutually compatible observables). [5 marks]

QUESTION THREE (15 MARKS)

a. Starting from the Klein-Gordon equation, obtain the equation of continuity. (5 marks)

b. Show that the Dirac matrices a*x*, a*y*, a*z* and b anticommute in pairs and their squares are unity.

(10 marks)

QUESTION FOUR (15 MARKS)

a. By considering the rate of change of the expectation value of $\hat{A}$,show that if $\hat{A}$ commutes with the hamiltonian $\hat{H}$, then $\hat{A}$ is conserved. (10 marks)

b. Proof that The precise relation for operators that do not have an intrinsic dependence on the time (in the sense that $\frac{\partial \Omega}{\partial t} = 0$) is, (5 marks)

$$\frac{d\langle \Omega \rangle}{dt} = \frac{i}{\hbar} \langle [H, \Omega] \rangle$$

QUESTION FIVE (15 MARKS)

Consider the Hamiltonian operator for a harmonic oscillator $(c=\hbar=1)$

$$\widehat{H}=\frac{1}{2m}p^2+\frac{1}{2}kx^2, k=m\omega^2$$

a. Define the operators (2 marks)

$$\hat{a}^{\dagger}=\frac{1}{\sqrt{2m\omega}}(\hat{p}+im\omega\,\hat{x}), \hat{a}=\frac{1}{\sqrt{2m\omega}}(\hat{p}-im\omega\,\hat{x})$$

b. Show that $\widehat{H}=\omega(a^{\dagger}a-\frac{1}{2})\hbar$ (3 marks)

c. Find the commutation relations for these operators using the definition of the hamiltonian in b above. (3 marks)

d. Using the definition of the Hamiltonian in b above, show that all expectation values of the Hamiltonian are positive definite, and in particular, all energies are positive.

(3 marks)

e. By first defining the Number operator N, show that the lowest energy state is $|0\rangle$ has the energy ½ $\hbar\omega$ i.e. $\widehat{H}|0\rangle=$ ½ $\hbar\omega|0\rangle$ (4 marks)

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