

**THIRD YEAR SECOND SEMESTER EXAMINATION FOR DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

ACMT 302 – ACTUARIAL MATHEMATICS II

DURATION: 2 HOURS

DATE:

TIME:

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a) Define the following

i) Pure premium (2 marks)

ii) Reserve (2 marks)

b) Given the values of l_x in the table below, what is ${}_4P_{50:51}$ (4 marks)

x	50	51	52	53	54	55	56
l_x	9,056	8,952	8,905	8,862	8,825	8,756	8,732

c) Draw a diagram for a multi-state model that could be used to value a life assurance policy where a sum assured is payable immediately on death and where premiums are waived during periods of sickness. Label all the states and transition intensities. (5 marks)

d) Differentiate between accrued benefit and future service benefit (4 marks)

e) On 1 October 2003 a pension scheme member was aged exactly 45, and had earned £40,000 over the previous year.

The salary scale s_x is defined such that for a life aged exactly x and for any integer $t > 0$:

$$\frac{s_{x+t}}{s_x} = \frac{\text{expected earnings between ages } x+t \text{ and } x+t+1}{\text{expected earnings between ages } x \text{ and } x+1}$$

Final salary is defined as the earnings received in the year immediately prior to retirement.

Salaries are increased on 1 January each year.

Write down and explain an expression for the expected final salary for this member, given that he intends to retire on 31 December following his 60th birthday. (7 marks)

f)

Calculate $p_{62:65}$ and ${}_3q_{50:50}$ using AM92 Ultimate mortality. (6 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a) When does the annuity $a_{70|75}$ pay? (2 marks)

b) A 25-year endowment assurance policy provides a payment of £75,000 on maturity or at the end of the year of earlier death. Calculate the annual premium payable for a policyholder who effects this insurance at exact age 45.

Expenses are 75% of the first premium and 5% of each subsequent premium, plus an initial expense of £250.

Assume AM92 Select mortality and 4% pa interest.(14 marks)

c) List the types of expenses incurred in writing a life insurance contract (4 marks)

QUESTION THREE

a) Define what is profit testing)(2 marks)

b) List the three categories of direct expenses allocated to policies (3 marks)

c) Describe the type of benefit provided by a unit-linked contract.(2 marks)

d) A life assurance policy is written for two lives aged 35 and 48 which pays £60,000 if the younger life dies first, £40,000 if the older life dies first and, in addition, pays £10,000 if both survive at least 25 years. Write a formula for the expected present value of this benefit.(5 marks)

e)) John, aged exactly 35, buys a term assurance policy that pays a benefit of £100,000 at the end of the year of his death if he dies before age 65. What is the expected accumulated value of the benefits at time 10?

Basis: AM92 Ultimate, 6% pa interest (8 marks)

QUESTION FOUR

a)

Explain $\ddot{a}_{xy}^{(12)}$ in words (2 marks)

b) A life insurance company is studying the profitability of a 5-year unit-linked endowment assurance contract. Details are as follows:

Age at inception	50
Annual premium	£2,000
Benefit	The greater of the bid value of units and £5,000 (paid at the end of the policy year)
Allocation	First year: 60% Other years: 98%
Bid/offer spread	5%
Management charge	1% (deducted at end of year)
Unit growth	6%
Interest for non-unit fund	6%
Mortality	AM92 Ultimate
Expenses	Initial £1,150 Renewal £75 at the start of the second year, subsequently inflating at 4% <i>pa</i>

Calculate the expected profit or loss on the non-unit fund in each year, per policy in force at the start of each year.(18 marks)

QUESTION FIVE

A five-year policy with annual cash flows issued to a life (x) produces the profit vector $Pr' = (-360.98, 149.66, 14.75, 273.19, 388.04, 403.00)$, where Pr_0 is the profit at time 0 and Pr_t ($t = 1, 2, \dots, 5$) is the profit at time t per policy in force at time $t - 1$.

The survival model used in the profit test is given by $Q_{x+t} = 0.0085 + 0.0005t$.

- Calculate the profit signature for this policy.(5 marks)
- Calculate the NPV for this policy using a risk discount rate of 10% per year.(5 marks)
- Calculate the NPV for this policy using a risk discount rate of 15% per year. .(5 marks)
- Comment briefly on the difference between your answers to parts (b) and (c). (2 marks)
- Calculate the IRR for this policy.(3 marks)

