

## FINANCIAL MATHEMATICS TWO

### FNMT 301

#### QUESTION ONE (30 MARKS)

- a. Bonds are classified as investments. The returns achieved over a period up to maturity date of the bond will vary. Give and explain four reasons why the returns of bonds vary? (2 marks)
- b. A tax-exempt investor purchases Kshs. 10,000 nominal of a newly issued 5 years fixed interest bond which is redeemable at par and it pays coupons of 8% p.a half yearly in arrears. Calculate the price the investor should pay to obtain a yield of 10% p.a.( 5 marks)
- c. Differentiate between contours time spot rates and continuous time forward rates and continuous time forward rates (4 marks).
- d. A fund must make payments of Kshs. 50,000/= at the end of sixth and eight years. Show that, if the interest rates are currently 7% p.a at all durations, immunization to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5 year zero coupon bond and 10 year zero coupon bond. (6 marks).
- e. List the advantages and disadvantages of issuing bonds at a fixed prices (4 marks).
- f. An index-linked bond pays half-yearly coupons and it's redeemable at per on 28/02/2024. The coupon paid n 28/02/2008 was Kshs. 2.10. A non-tax payer buys a nominal of the bond on 29/02/2008. Assume future inflation is 5.25% p.a, how much should the investor pay in order to obtain the money rate of return of 10% p.a. (6 marks).
- g. State and explain three limitations of immunization. (3 marks)

**QUESTION TWO (20 MARKS)**

- a. Consider a 3 ½ % coupon stock issued in February 2010 and redeemed in February 2015. The coupon payments are made each year and are linked to an inflation index with a one year term lag. The index value of each February from 2009 and 2015 are given in the table below.

Year	2009	2010	2011	2012	2013	2014	2015
Index	540	562	584	607	632	657	788

The base month of indexation is February 2009 because a time lag. Find the inflation rates implied by the table of indices. (15 marks)

- b. Differentiate between a stochastic rate model and a varying rate model. Give the formulas of single premium between a varying interest rate model and fixed rate model. (5 marks)

**QUESTION THREE (20 MARKS)**

- a. A 10 years bond with half yearly coupons at 6% has just been issued with a redemption yield of 9% p.a. It is redeemable at par. What price would an investor paying 15% tax on income pay for the bond? Tax payments are due four months after each coupon is received. (5 marks).
- b. List and explain three securities with uncertain incomes ( 6 marks).
- c. The prices for a zero coupon bonds of various term as follows:

1 year	=	Kshs. 954%
5 years	=	Kshs. 70%
10 years	=	Kshs. 47%
15 years	=	Kshs. 30%

Kshs. x% means Kshs. x per Kshs. 100 nominal.

Calculate the spot rates for these terms. (5 marks)

- d. Differentiate between Expectation Theory and Liquidity Preference. (4 marks)

#### QUESTION FOUR (20 MARKS)

- a. Interest rates are fixed at either 4% p.a with probability 0.2 or 5% p.a with probability 0.8. What is the standard deviation of the present value of a payment of Kshs. 25,000/= in 5 years time (5 marks)
- b. A lump sum of Kshs. 14,000 will be invested at time 0 for 4 years at a constant annual rate of interest  $i(1+i)$  has log normal distribution with mean 1.05 and variance 0.007. What is the probability that the investment will accumulate to more than Kshs. 20,000/= in four years time. (5 marks)
- c. List and explain assumptions of mean variance portfolio theory (5 marks)
- d. List and explain types of investment related risk. (5 marks).

#### QUESTION FIVE (20 MARKS)

- a. Using mean variance portfolio theory, prove that the efficient frontier becomes a straight line in the presence of a risk free asset.  
Consider two independent assets, asset A and asset B, with expected return of 6% p.a. and 11% p.a and standard deviation of returns of 5% p.a. and 10% p.a respectively.
  - i. If only assets A and B are viable calculate the equation of the efficient frontier in expected return standard deviation space.
  - ii. A third asset, asset C, is risk free and has an expected return of 4% p.a. A Lagrangian function is to be used to calculate the equation of the new efficient frontier. Write down simultaneous equation that results from the procedure.
  - iii. Use the simultaneous equation to derive relationship between  $X_A$  and  $X_B$  the holdings an asset A and B, on new efficient frontier.
  - iv. Hence derive the equation of the new efficient frontier in expected return standard deviation (20 marks).

