CHUKA UNIVERSITY

UNIVERSITY EXAMINATION
RESIT/SUPPLEMENTARY / SPECIAL EXAMINATIONS
EXAMINATION FOR THE AWARD OF BACHELOR OF EDUCATION (SCIENCE/ARTS)

MATH 422: ORDINARY DIFFERENTIAL EQUATIONS II

STREAMS: BED (SCI/ARTS) **TIME: 2 HOURS**

DAY/DATE: WENESDAY 03/11/2021 **2.30 P.M - 4.30 P.M.**

INSTRUCTIONS

- **Answer Question <u>ONE</u> and any other <u>TWO</u> Questions**

<u>Question One (Compulsory) (30marks)</u>

a. Given the equation. $\left(1-x^2\right)\frac{d^2y}{dx^2} -{}_1 \frac{dy}{dx} + y = 0$, determine:

   i. Ordinary point of the equation (3 marks)

   ii. Singular point of the equation (3 marks)

   iii. Regular singular point of the equation (3 marks)

b. Find the differential equation whose fundamental set of solution is $\left\{e^{4x}, \quad e^{x}\right\}$

   (7 marks)

c. Show that the functions $f_1 = e^{x}$, $f_2 = e^{-x}$ and $f_3 = e^{3x}$ are

   i. Linearly independent (6 marks)

   ii. Write down the general solution of the differential equation for which they are solutions

   (2 marks)

d. Prove that the Legendre Polynomial of order three is given by

   $P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right)$ (6 marks)

Question Two (30 marks)

a. Find the power series solution of $y'' + 2xy = 0$ near $x = 0$ (10 mark)

b. Use the Rodrigues formula for Legendre's polynomial of order n

$$P_n(x) = \frac{1}{2^n\, n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$ find:

i. $P_2(x)$ (4 marks)

ii. $P_4(x)$ (6 marks)

Question Three (30 marks)

a. Find a power series solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{xdy}{dx} - 2y = 0$$ (12 marks)

b. Convert the differential equation. $\frac{2d^3y}{dt^3} + \frac{3d^2y}{dt^2} - \frac{4dy}{dt} + 5y = 0$ into a matrix equation of the form $\vec{\mathbf{y}}' = A\vec{\mathbf{y}}$ (8 marks)

Question Four (20 marks)

a. Show that the functions x , x^2 *and* x^3 are solutions to a differential equation and write the general solution (8 marks)

b. Use the method of undetermined coefficients to find the general solution of the nonhomogenous differential equation (12 marks)

$$\dot{x}_1 = x_2 + 2$$
$$\dot{x}_2 = -2x_1 + 3x_2 + 1$$

Question Five (20 marks)

a. Find the general solution of the system using the matrix method (12 marks)

$$\dot{y} = 2y_1 - 3y_2$$
$$\dot{y} = y_1 - 2y_2$$

b. The differential equation has a regular singular point at $x = 0$.Find the indicial equation and the recurrence formula (8 marks)

$$x^2 y'' - xy' + (1-x)y = 0$$

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