CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE/ARTS)

MATH 420/421: PARTIAL DIFFERENTIAL EQUATIONS 1

STREAMS: BSC   **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 03/11/2021   **2.30 P.M – 4.30 P.M.**

INSTRUCTIONS:
- **Answer Questions ONE (compulsory) and any other TWO Questions.**

QUESTION ONE (COMPULSORY) (30 MARKS)

a. Define a Pfaffian equation in:
   i.   2 dimension   (2marks)
   ii. 3dimension   (3Marks)

b. Find the differential equation arising from $z = ax + by + cxy$ (6marks)

c. Solve the Lagrange's equation $\dfrac{dx}{x} = \dfrac{dy}{y} = \dfrac{dz}{z}$ (7marks)

d. Eliminate the arbitrary constant from $z = (x-a)^2 + (y-b)^2$ (6marks)

e. Solve $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial u}{\partial x}$ (6marks)

QUESTION TWO (20 MARKS)

a. Find the orthogonal trajectories on the surface $x^2 + y^2 + 2gx + c = 0$ where g is a parameter and c is a constant (10 marks)

b. Eliminate the arbitrary constants a and b from the equation $ax^2 + by^2 + ab = z$ (4 marks)

c. Find the integral curves of the equation $\dfrac{dx}{y(x+y)+cz} = \dfrac{dy}{x(x+y)-cz} = \dfrac{dz}{z(x+y)}$ (6 marks)

QUESTION THREE (20 MARKS)

a. Solve the Type II Partial differential equation (8Marks)

b. Verify that the differential equation $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$ is integrable (5Marks)

c. Solve the Pfaffian equation $z\,dx + dy + dz = 0$ (7Marks)

QUESTION FOUR (20 MARKS)

a. Find the partial differential equation arising from $z = (x+a)(y+b)$ (8marks)

b. Solve the nonlinear partial differential equation given by $z = px + qy + 3p^{\frac{1}{3}}q^{\frac{1}{3}}$ for singular solution. (12Marks)

QUESTION FIVE (20 MARKS)

a. Verify that $u = f(x - ct) + g(x + ct)$ is a solution to $c^2 U_{xx} = U_{tt}$ (8marks)

b. Find the integral curves of the equation $\dfrac{dx}{x+y-z} = \dfrac{dy}{x+y-z} = \dfrac{dz}{2(z-y)}$ taking $1, -3, -1$ as multipliers (12marks)