

CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATIONS FOR THE AWARD OF BACHELOR OF SCIENCE IN APPLIED
COMPUTER SCIENCE.

MATH 407: FOURIER ANALYSIS

TIME: 2 HOURS

INSTRUCTIONS

Answer question one and any other two questions

Adhere to the instructions on the answer booklet.

QUESTION ONE Compulsory.

- a. Find the complex Fourier series for $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ 2, & 0 \leq x \leq \pi \end{cases}$ and show that $c_n = \begin{cases} \frac{1}{in\pi}, & n \text{ is odd} \\ 0, & n \text{ is even} \end{cases}$

5mks

- b. Evaluate $\int_0^1 (1-x^3)^{-\frac{1}{2}} dx$ using the Beta function

5mks

- c. Using the Fourier cosine integral representation of an appropriate function, show that

$$\int_0^{\infty} \frac{\cos wx}{k^2 + w^2} dw = \frac{\pi e^{-kx}}{2k}$$

5mks

- d. Obtain the Fourier sine transform of $f(x) = \frac{e^{-3x}}{x}$

5mks

- e. Determine the exponential form of the Fourier series for the function defined by $f(t) = 2t, -\pi \leq t \leq \pi$

and show that the Fourier series is equivalent to $f(t) = 4 \left(\sin t - \frac{1}{2} \sin 2t + \frac{1}{3} \sin 3t - \frac{1}{4} \sin 4t + \dots \right)$

5mks

- f. Evaluate $\int_0^{\infty} x^{n-1} e^{-4x^2} dx$ by the Gamma function

5mks

QUESTION TWO

- a. State the Dirichlets conditions for a Fourier series

5mks

- b. Find the Fourier series representing $f(x) = x, 0 < x < 2\pi$

7mks

- c. Obtain a_0, a_n and b_n for the periodic function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ 8mks

QUESTION THREE

- a. A periodic function of period 4 is defined as $f(x) = \begin{cases} x, & 0 < x < 2 \\ -x, & -2 < x < 0 \end{cases}$
Obtain a_0, a_n and b_n 8mks

- b. Use the Fourier sine series for $f(x) = 1$, in $0 < x < \pi$ to show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \dots$ 7mks

- c. Evaluate $\int_0^{\infty} 4\sqrt{x}e^{-\sqrt{x}} dx$ 5mks

QUESTION FOUR

- a. Find the Fourier cosine transform of
 $f(x) = e^{-2x} + 4e^{-3x}$ 6mks

- b. Solve for $f(x)$ from the integral equation $\int_0^{\infty} f(x) \cos sx dx = e^{-s}$ 7mks

- c. Use Parsaval's identity to show that $\int_0^{\infty} \frac{dx}{(x^2 + 1)^2} dt = \frac{\pi}{4}$ 7mks

QUESTION FIVE

- a. Solve $U_t = kU_{xx}$ for $x \geq 0, t \geq 0$, under the given conditions $U = U_0$ at $x = 0, t > 0$, with initial conditions $U(x, 0) = 0, x \geq 0$ by Fourier transforms. 8mks

- b. Find the finite Fourier sine and cosine transform of $f(x) = x$, in $(0, l)$ 6mks

- c. Find the finite Fourier sine transform of $f(x) = 1$ in $(0, \pi)$ and use the inversion theorem to prove that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8} \quad \text{6mks}$$