

CHUKA



UNIVERSITY

## UNIVERSITY EXAMINATION

RESIT/SUPPLEMENTARY / SPECIAL EXAMINATIONS EXAMINATION FOR THE  
AWARD OF DEGREE IN BACHELOR OF

MATH 221: CALCULUS II

STREAMS:

TIME: 2 HOURS

DAY/DATE: MONDAY 3/5/2021

8.30 A.M - 10.30 P.M.

INSTRUCTIONS: Answer question one (compulsory) and any other two questions

## Question One (Compulsory) (30 marks)

a) Evaluate

i.  $\int \frac{1}{\sqrt{2x+1}} dx$  (3 marks)

ii.  $\int e^x \cos x dx$  (5 marks)

b) i. Approximate  $\int_0^1 5x^4 dx$  using Simpson's rule with  $n = 4$ . (3 marks)

ii. What is the maximum error in the approximation in (i) above? (2 marks)

c) Evaluate:

$\int \frac{4-2x}{(x^2+1)(x-1)^2} dx$  (5 marks)

d) Use the reduction formula  $I_n = \frac{n-1}{n} I_{n-2}$ ,  $n \geq 2$  where  $I_n = \int_0^{\pi/2} \cos^n x dx$  to

Evaluate  $\int_0^{\pi/2} \cos^{10} x dx$  (4 marks)

e) Evaluate

i.  $\int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx$  (3 marks)

ii.  $\int \frac{x^2}{(x^2+9)^2} dx$  (2 marks)

**Question Two (20 marks)**

- a) Determine the area bounded by the curves  $y^2 = x$  and  $y = x - 2$  (6 marks)
- b) Evaluate
- $\int_{-3/4}^{3/4} \frac{dx}{\sqrt{9-4x^2}}$  (5 marks)
  - $\int \frac{x^4 - x^3 + x^2 - x + 1}{x^3 + x} dx$  (6 marks)
  - $\int x^2 e^{x^3 - 5} dx$  (3 marks)

**Question Three (20 marks)**

- (a) Find the surface area of a sphere obtained by rotating  $x^2 + y^2 = a^2$  between  $x = a$  and  $x = -a$  and the  $x$ -axis. (5 marks)
- (b) Evaluate  $\int x^3 \sqrt{4 - x^2} dx$  using trigonometric substitution (5 marks)
- (c) i. Show that  $\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$  where  $n = 2, 3, \dots$  (5 marks)
- ii. Hence evaluate  $\int_0^{\pi/2} \sin^4 x dx$ . (5 marks)

**Question Four (20 marks)**

- (a) Determine the length of the arc of the graph whose equation is  $6xy - y^4 - 3 = 0$  from  $(19/12, 2)$  to  $(14/3, 3)$  (6 marks)
- (b) Evaluate  $\int \sin^{-1} 4x dx$ . (5 marks)
- (c) Evaluate  $\int \sin^3 x \cos^2 x dx$  (5 marks)
- (d) Evaluate  $\int_1^e \frac{dx}{x(1+\ln x)}$  (4 marks)

**Question Five (20 marks)**

- a) i. Approximate  $\int_2^3 \sqrt{1 + x^3} dx$  using Trapezoidal rule with  $n = 4$ . (3 marks)
- ii. Determine the maximum error in the approximation. (2 marks)
- b) Evaluate  $\int \frac{4x^3 + 4x + 3}{(x^2 + 1)^2} dx$  (7 marks)
- c) Evaluate  $\int \frac{dx}{1 + \tan x}$  using the substitution  $u = \tan x$ . (4 marks)
- d) Evaluate  $\int \frac{dx}{(16x^2 - 25)^{3/2}}$  using hyperbolic substitution (4 marks)