CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

MATH 223: OPTIMIZATION

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: FRIDAY 05/11/2021 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS:

- **Answer ALL questions**
- **Adhere to the instructions on the answer booklet.**

QUESTIONS ONE

a. Define the following terms

i.	Operations Research	(2 marks)
ii.	Deterministic models.	(2 marks)
iii.	Probabilistic (or Stochastic) models.	(2 marks)
iv.	Objective Function	(2 marks)

b. Sketch the following constraints by clearly indicating all intercepts with the axes and the feasible region (4 marks)

$2 \le x \le 6$; $y \ge 1$; $3x+2y \ge 12$; $9y+7x \le 63$

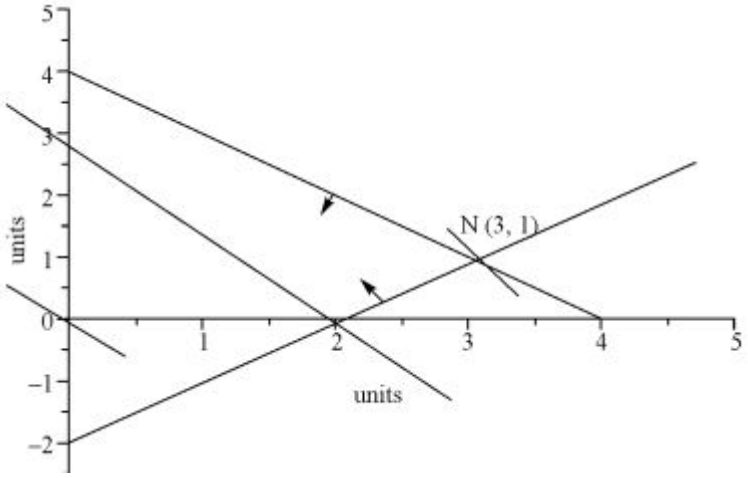
Use the objective function $P = 3x + 2y$ to maximise P with respect to the feasible region.

c. Solve the L.P. problem below .

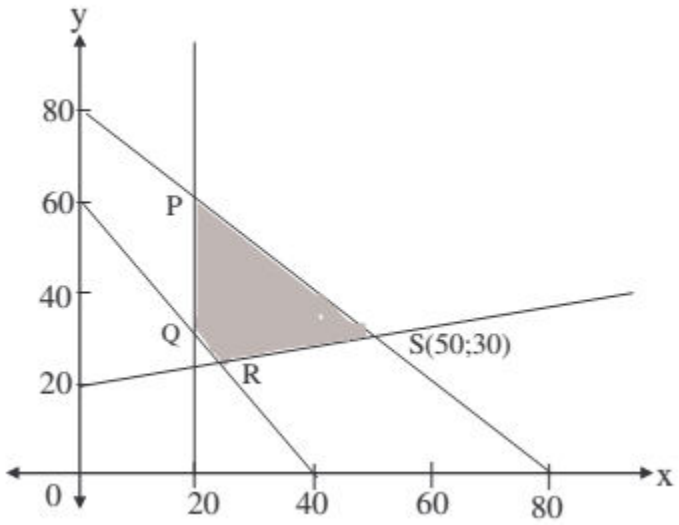
Maximise $Z = 3a + 2b$ S.T.

$1a + 1b \le 4$

$1a - 1b \le 2$ and both a and b are ≥ 0 (6 marks)



d. In the accompanying sketch there is a set of inequalities that leads to the feasible region PQRS as shown by the shaded area. Use the graph to answer the following questions :



i. Write down the set of inequalities that is represented by the feasible region (5 marks)

ii. Maximise 3x + 2y for the given feasible region. (4 marks)

iii. The co-ordinates of point R Minimise the function value of k in y = mx + k . Write down the possible values of m. (3 marks)

QUESTION TWO

a. A company manufactures two products *X* and *Y*. The profit contribution of *X* and *Y* are ksh.3/- and ksh. 4/- respectively. The products *X* and *Y* require the services of four facilities. The capacities of the four facilities *A*, *B*, *C*, and *D* are limited and the available capacities in hours are 200 Hrs, 150 Hrs, 100 Hrs and 80 hours respectively. Product *X* requires 5, 3, 5 and 8 hours of facilities *A*, *B*, *C* and *D* respectively. Similarly the requirement of product *Y* is 4, 5, 5, and 4 hours respectively on *A*, *B*, *C* and *D*. Find the optimal product mix to maximise the profit. (6 marks)

b. A furniture manufacturer produces two types of display cabinets, type X and type Y. On a weekly basis he must produce at least 2 of each type, but not more than 5 of type X or more than 6 of type Y. It takes 4 hours to produce type X and 5 hours for type Y in a 40 hour working week. At least 12 workers are needed with 2 working on type X and 3 on type Y at any one time.

i. Represent the above information as a system of inequalities . (4 marks)

ii. If the profit (P) on type X is ksh800 and on type Y is ksh1000, write down the objective function in the form P = ax + by . (2 marks)

c. A patient consult a doctor to check up his ill health. Doctor examines him and advises him that he is having deficiency of two vitamins, vitamin *A* and vitamin *D*. The Doctor advises him to consume vitamin *A* and *D* regularly for a period of time so that he can regain his health. The Doctor prescribes tonic *X* and tonic *Y*, which are having vitamin *A*, and *D* in certain proportion. Also advises the patient to consume **at least** 40 units of vitamin *A* and 50 units of vitamin Daily. The cost of tonics *X* and *Y* and the proportion of vitamin *A* and *D* that present in *X* and *Y* are given in the table below.

Vitamins	*Tonics*		*Daily requirement in units.*
	X	*Y*	
A	2	4	40
D	3	2	50
Cost in Rs. per unit.	5	3	

Formulate a LP. problem to minimize the cost of tonics and find the minimum costs (8 marks)

QUESTION THREE

a. A retail store stocks two types of shirts *A* and *B*. These are packed in attractive cardboard boxes. During a week the store can sell a maximum of 400 shirts of type *A* and a maximum of 300 shirts of type *B*. The storage capacity, however, is limited to a maximum of 600 of both types combined. Type *A* shirt fetches a profit of . 2/- per unit and type *B* a profit of 5/- per unit. How many of each type the store should stock per week to maximize the total profit? Formulate a mathematical model of the problem. (5 marks)

b. Maximize $y = 0.05x + 0.09y + 0.08z$, subject to the following constraints: (5 marks)

$$x + y + z \le 150$$
$$x \le 75$$
$$y \le 75$$
$$z \le 75$$

c. Minimize: f = 32x +12y subject to the following constraints: (5 marks)

$$4x + 3y \ge 6$$
$$8x + 2y \ge 5$$

with x and y nonnegative.

e. Maximize $P = 3x + 4y + z$ subject to:

$$\begin{aligned} x + 2y + z &\le 6 \\ 2x + \quad 2z &\le 4 \\ 3x + y + z &\le 9 \\ x,\ y,\ z &\ge 0 \end{aligned}$$

(5 marks)
