CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, BCHELOR OF EDUCATION AND BACHELOR OF ARTS

MATH 241 : PROBABILITY AND STATISTICS 1

STREAMS: BSC, BED & BA **TIME: 2 HOURS**

DAY/DATE: MONDAY 01/11/2021 **11.30 A.M. – 1.30 P.M.**

INSTRUCTION:

- **Answer ALL the Questions.**

Question One (30 marks)

a) Distinguish between the following probability mass function and a probability density function (2 marks)

b) A discrete random variable has the following p.m.f.

x	3	4	5	6	7
$Pv(X = x)$	0.1	a	0.3	b	0.2

If $E(x) = 5.2$.

Find the

i) values of 'a' and 'b' (4 marks)

ii) The Var(x) (3 marks)

iii) Cdf of the random variable (2 marks)

c) A random variable has the exponential distribution

$$f(x) = \begin{cases} \lambda e^{-\lambda x}; & x > 0 \\ 0 & otherwise \end{cases}$$

i) Show that the moment generating function is

$$M_x(t) = \left(1 - {}^{t}/_{\lambda}\right)^{-1}$$ (4 marks)

ii) Hence find the mean and variance of x. (7 marks)

d) If $f(x) = b(x: n, p), E(x) = 4, \text{var}(x) = 3$. Compute $\Pr(x \geq 1)$ (3 marks)

e) A random variable x is normally distributed with $\mu = 20$ and $\sigma^2 = 16$. If

$$Y = \frac{1}{4}(x - 20)$$

Find

i) $E(Y)$

ii) Var (Y)

iii) Hence or otherwise write down the p.d.f. of Y. (5 marks)

Question Two (20 marks)

a) Let Y be a continuous random variable. Show that the function

$$f(y) = \begin{cases} \frac{y}{2} & 0 \leq y \leq 2 \\ 0 & otherwise \end{cases}$$

Is indeed a probability density function.

(2 marks)

Hence determine

i) $\Pr(-1 \leq y \leq 1)$ (3 marks)

ii) $E(y)$ (3 marks)

iii) The cumulative distribution function $F(y)$

(3 marks)

b) Give $f(x)$ is a Poisson distribution function of r.v x, show that it's moment generating function is given by $m_x(t) = e^{\lambda(e^t - 1)}$. Hence or otherwise determine the mean of x.

(9 marks)

Question Three (20 marks)

a) The annual number of earthquakes registering at least 2.5 on the Richter Scale and having an epicenter within 40 miles of downtown Memphis follows a Poisson distribution with mean 6.5. What is the probability that at least 9 such earthquakes will strike next year? (5 marks)

b) Let X be a Bernoulli random variable with probability mass function given by;

$$\begin{cases} p^x(1-p)^{1-x}, x = 0,1 \\ 0 \qquad\qquad\qquad , otherwise \end{cases}$$

Find;

i) The factorial moment generating (5 marks)

ii) The probability generating function (5 marks)

iii) The mean and variance (5 marks)
