CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE, ART, EDUCATION

MATH 242: PROBABILITY AND STATISTICS II

STREAMS: BSC, BED & BA **TIME: 2 HOURS**

DAY/DATE: TUESDAY 10/08/2021 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS

- Answer Question **ALL** the THREE Questions.

QUESTION ONE [30 MARKS]

a) Let x and y be random variables with a joint density function.

$$f(x,y) = \begin{cases} 4xy\,, 0 \le x,y \le 1 \\ 0\ , \ elsewhere \end{cases}$$

Find

(i) $Cov(x,y)$ [8 marks]

(ii) P(y/x=$\frac{1}{2}$) [4 marks]

b) Suppose that X and Y be two continuous random variables with joint density function

$$f(x,y) = f(x) = \begin{cases} \frac{k}{2}x^3y^3, & 0 \le x \le 2, 0 \le y \le 2 \\ 0, & Otherwise \end{cases}$$

i. Find the value of k [3 marks]

ii. Determine whether variables X and Y are independent [4marks]

iii. Find=$P\left(X \le \frac{1}{2} \therefore Y > \frac{1}{2}\right)$ [3marks]

iv. Find Var (X+2Y) [3marks]

c) Let X and Y have the joint density function

$$f(x,y) = \begin{cases} 4xy, & 0 \le x \le k, 0 \le y \le 1 \\ 0, & Otherwise \end{cases}$$

Find

i. Show that the value of k is 1 [2 marks]
ii. The conditional P D F of Y given $X = x$ [3 marks]

QUESTION TWO (20 MARKS)

a) The joint probability density function of X and Y is represented by the table below;

Y / X	1	2	3	4
1	0.05	0.75	0.065	0.093
2	0.059	0.1	0.12	0.102
3	0.05	0.001	0.082	0.003
4	0.1	0.07	0.01	0.02

Find (i) the marginal probability distributions of X and Y [3 marks]
(ii) the conditional probability of X given Y=3 [3 marks]
(iii) E(X/Y=3) and Var(X/Y=3) [6 marks]

b) Given the joint p.d.f of X and Y is

$$f(x,y) = \begin{cases} x\left(x + \frac{1}{3}y\right), 0 \le x \le 1, 0 \le y \le 2 \\ 0 \quad , \quad elsewhere \end{cases}$$

Find (a) (i) E(X) (ii) E(Y) (iii) E(XY) [6 marks]
(b) Are X and Y independent? [2 marks]

QUESTION THREE (20 MARKS)

a) The joint p.d.f of X and Y is $f(x,y) = \begin{cases} 2e^{-x-y}, & 0 < x < y, 0 < y < \infty \\ 0, & elsewhere \end{cases}$

Find
(i) the joint moment generating function of X and Y. [8 marks]
(ii) E(X) [4 marks]

b) Consider a sample of size two draw without replacement from an urn containing three balls numbered 1,2 and 3. Let X be the smaller of the two numbers drawn and Y the larger of the two numbers drawn.
Find
i. the joint probability distribution function of X and Y
ii. the conditional distribution of Y given X=1
iii. Cov(X,Y) [8 marks]
