CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/ SPECIAL EXAMINATIONS

EXAMINATION FOR THE AWARED OF BACHELOR OF SCIENCE

MATH 301/316: LINEAR ALGEBRA II

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 11/8/2021 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS: Answer Question ONE and Any Other TWO Questions

QUESTION ONE: (30 MARKS)

a) Given that $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & -4 & 3 \\ 6 & -6 & 5 \end{bmatrix}$, find the eigenvalues of A^3 (6 marks)

b) Find the symmetric matrix that correspond to the following quadratic form
$q(x, y, z) = 2x^2 - 8xz + y^2 - 16xz + 14yz + 5z^2$ (4 marks)

c) Prove that similar matrices have the same characteristic polynomial. (4 marks)

d) State how elementary row operations affect the determinant of a square matrix (3 marks)

e) Show that if $A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}$, then A is a zero of the function $f(t) = t^2 - 5t + 7$ (3 marks)

f) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 3 & 2 \\ 0 & -1 & 0 \end{bmatrix}$ (5 marks)

g) State Cayley-Hamilton theorem and verify using a linear operator $T : R^2 \to R^2$ defined by $T(x, y,) = (x - 3y, 2x + 5y)$ (5 marks)

QUESTION TWO (20 MARKS)

a) Let f be a bilinear form on R^2 defined by $f[(x_1,x_2),(y_1,y_2)] = 2x_1y_1 - 3x_1y_2 + 4x_2y_2$. Find

   i. The matrix A of f in the basis $\{u_1 = (1,0), u_2 = (1,1)\}$

   ii. The matrix B of f in the basis $\{v_1 = (2,1), v_2 = (1,-1)\}$

   iii. The change of basis matrix P from the basis $\{u_i\}$ to the basis $\{v_i\}$ and verify that $B = P^T AP$. (12 marks)

b) Let A be the matrix

$$\begin{bmatrix} 1 & -3 & 2 \\ -3 & 4 & -5 \\ 2 & -5 & 8 \end{bmatrix}$$

Apply diagonalization algorithm to obtain a matrix P such that $D = P^T AP$ (8 marks)

QUESTION THREE (20 MARKS)

a) Apply Gram-Schmidt orthogonalization process to the basis $B = \{1, x, x^2\}$ in $P_2(x)$ with the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x)q(x)dx$ to obtain an orthogonal set (7 marks)

b) Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$

   i. Find the characteristic polynomial of A. (2 marks)

   ii. Find all the eigenvalues λ_1 and λ_2 of A and their corresponding eigenvectors. (4 marks)

   iii. Is A diagonalizable? If yes, Determine the matrices P and D such that $D = P^{-1}AP$ such that D is diagonal. (2 marks)

   iv. Consider a polynomial $f(x) = x^3 - 5x^2 + 3x + 6$. Find $f(\lambda_1)$ and $f(\lambda_2)$,hence find $f(A)$ (3 marks)

   v. Find a matrix B such that $B^2 = A$ (2 marks)
