CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (GENERAL), ECON/MATH

MATH 304: COMPLEX ANALYSIS 1

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 08/07/2021   **11.30 A.M – 1.30 P.M**

INSTRUCTIONS
Answer Questions ONE (compulsory) and any other TWO Questions

QUESTION ONE (30 MARKS) COMPULSORY

a. Simplify and write the complex expression in the standard form $a+bi$.   (1,3,3 Marks)

  i. $\dfrac{1-i}{2}$

  ii. $(\overline{2-i})^2$

  iii. $\left(\dfrac{1}{2}+\dfrac{i}{7}\right)\left(\dfrac{3}{2}-i\right)$

b. If $w=f(z)=\dfrac{1+z}{1-z}$

  i.   Determine the point where $f(z)$ is not analytic   (2Marks)

  ii. Find $\dfrac{dw}{dz}$ (3Marks)

c. Evaluate the following Limits

  i. $\displaystyle\lim_{z\to 1-i}(z^2-5z+10)$ (3Marks)

  ii. $\displaystyle\lim_{z\to -2i}\dfrac{(2z+3)(2-1)}{(z^2-2z+4)}$ (4Marks)

d. Show that is $f(\ )=-2xy+i(x^2-y^2)$ analytic   (5Marks)

e. Convert the given Complex number into the form indicated

i. $\sqrt{3}-i$ into polar form   (3Marks)

ii. $2\left(Cos120^\circ + iSin120^\circ\right)$ into Cartesian form   (3Marks)

QUESTION TWO (20 MARKS)

a. Solve the following for z

    i. $(2+3i)z=(2-i)z-i$ (3Mark)

    ii. $iz+2i=4$ (4Mark)

b.   i. State the Cauchy Integral formula   (2marks)

    ii. Using the Cauchy Integral formula evaluate $\oint_C \dfrac{z^2-4z+4}{z+i}dz$, where C is the circle $|z|=2$ (5Marks)

c. Compute the Laurent series for the function $f(z)=\dfrac{z+1}{z^3\left(z^2+1\right)}$ on the region $A: 0<|z|<1$ centered at $z=0$.   (6Marks)

QUESTION THREE (20 MARKS)

a. Simplify $\dfrac{14+3i}{2-i}$ and give your answer in the form $x+iy$ (4Marks)

b. Find all the residues of $f(z)=\dfrac{1}{z^2+2z+10}$ (9Marks)

c. Using DeMoivre's Theorem $\left(Cos\theta + iSin\theta\right)^n = Cosn\theta+iSinn\theta$, show that

$$Tan3\theta = \frac{3Tan\theta - Tan^3\theta}{1-3Tan^2\theta}$$ (7Marks)

QUESTION FOUR (20 MARKS)

a. Solve the equation $z^2 + 4z + 5 = 0$ (3Marks)

b. Evaluate the integral using the residue theorem with $|z| = 3$ (13Marks)

c. Find $f(z) = u + iv$, given that $f(z)$ is analytic and $u = x^3 - 3x^2 y$ (4Marks)

QUESTION FIVE (20 MARKS)

a. State DeMoivre's Theorems on:

    i.   Powers of complex numbers (2Marks)

    ii. n^{th} roots of complex numbers (2Marks)

b. Using the Theorems stated in 5a above:

    i.   Expand $z = (1 + i)^9$ (4Marks)

    ii.   Find the square roots of $z = 2 + i2\sqrt{3}$ (5Marks)

c. Determine the region of the w-plane into which the region bounded

    $x = 1. y = 1 \ and \ x + y = 1$ by is mapped by the transformation $w = z^2$ (7Marks)