CHUKA   **UNIVERSITY**

UNIVERSITY EXAMINATION

RESIT/SPECIAL EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 313/303: REAL ANALYSIS II

STREAMS: **TIME: 2 HOURS**

DAY/DATE: TUESDAY 04/05/2021   **11.30 A.M – 1.30 P.M**

INSTRUCTIONS:

- Answer question **ALL** the questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a)   (i) When is the sequence x_n of elements of real or complex numbers said to be convergent?

(2 marks)

        (ii) Let x_n, y_n and z_n be sequences of real numbers such that $x_n \leq z_n \leq y_n$ $\forall\, n \geq N$ (N is a fixed integer). Let x_n, y_n both converge to the same limit, say l. Show that z_n also converges to l $as\ n \to \infty$ (5 marks)

(b) Take $a, b > 0$ $(a, b \neq 1)$, prove that $log_a x = \frac{log_b x}{log_b a}$ (3 marks)

© Define and give an example of a periodic function   (2 marks)

(d)   (i) Define an absolutely convergent series   (2 marks)

        (ii) Show that in general absolute convergence implies convergence in (K, d) (3 marks)

(e)   (i) Let $\sum_{k \in N} x_k$ be a series of real numbers. Prove that if $|x_k| \leq y_k \ \forall \ k \in N$ and $\sum_{k \in N} y_k$ is convergent, then the sum $\sum_{k \in N} x_k$ is also convergent   (4 marks)

    (ii) Prove that if $p = 1$, then the series $\sum_{n \in N} \frac{1}{n^P}$ is divergent   (5 marks)

(f) Define the Fourier series of the function $f(x)$ on the interval $-l$ to l (4 marks)

QUESTION TWO: (20 MARKS)

(a) (i) Write the general expression of an exponential and logarithmic function whose base is a
      (2marks)

(ii) By considering $a > 1 \ and \ 0 < a < 1$ for the functions f(x) and g(x) respectively make a comparison of the exponential and logarithmic functions. Hence state any three differences in these graphs.   (8marks)

(b) Find the Fourier series of the function defined by

$$f(x) = 0, \quad for -\pi < x < 0, \quad and \quad f(x) = x \quad for \ 0 < x < \pi$$

      (10marks)

QUESTION THREE: (20 MARKS)

(a) (i) Describe the Riemann Integrable function f on the interval $[a, b]$ (4 marks)

(ii) Show that a Dirichlet function on the interval $[a, b]$ is not Riemann Integrable.   (6 marks)

(b) Show that the function $f(x) = x$ is Riemann Integrable in $[0,1]$ that $\int_0^1 f(x) = \frac{1}{2}$ (10 marks)