CHUKA UNIVERSITY

UNIVERSITY EXAMINATION

RESIT/SUPPLEMENTARY / SPECIAL EXAMINATIONS FOR THE AWARD OF DEGREE IN BACHELOR OF

MATH 316/301: LINEAR ALGEBRA II

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 06/05/2021 **8.30 A.M - 10.30 A.M.**

INSTRUCTIONS: ANSWER ALL THE QUESTIONS

QUESTION ONE: (30 MARKS)

a) Given that $A = \begin{bmatrix} 2 & -3 & 3 \\ 3 & -4 & 3 \\ 6 & -6 & 5 \end{bmatrix}$, find the eigenvalues of A^{-1} (6 marks)

b) Find the symmetric matrix that correspond to the following quadratic form
$q(x, y, z) = 2x^2 - 8xz + y^2 - 16xz + 14yz + 5z^2$ (4 marks)

c) Prove that similar matrices have the same characteristic polynomial. (4 marks)

d) State how elementary row operations affect the determinant of a square matrix (3 marks)

e) Show that if $A = \begin{bmatrix} 1 & -1 \\ 3 & 4 \end{bmatrix}$, then A is a zero of the function $f(t) = t^2 - 5t + 7$ (3 marks)

f) Find the minimal polynomial of the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 3 & 2 \\ 0 & -1 & 0 \end{bmatrix}$ (5 marks)

g) State Cayley-Hamilton theorem and verify using a linear operator $T : R^2 \rightarrow R^2$ defined by $T(x, y,) = (x - 3y, 2x + 5y)$
(5 marks)

QUESTION TWO (20 MARKS)

a) Let f be a bilinear form on R^2 defined by $f[(x_1, x_2), (y_1, y_2)] = 2x_1y_1 - 3x_1y_2 + 4x_2y_2$. Find

i. The matrix A of f in the basis $\{u_1 = (1,0), u_2 = (1,1)\}$

ii. The matrix B of f in the basis $\{v_1 = (2,1), v_2 = (1,-1)\}$

iii. The change of basis matrix P from the basis $\{u_i\}$ to the basis $\{v_i\}$ and verify that $B = P^T AP$. (12 marks)

b) Let A be the matrix

$$\begin{bmatrix} 1 & -3 & 2 \\ -3 & 7 & -5 \\ 2 & -5 & 8 \end{bmatrix}$$

Apply diagonalization algorithm to obtain a matrix P such that $D = P^T AP$ (8 marks)

QUESTION THREE (20 MARKS)

a) Given that $A = \begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 5 & -3 & 1 \\ 1 & 6 & -2 & 4 \\ 0 & 1 & -3 & 7 \end{bmatrix}$ determine the number n_k and the sum S_k of principal minors of order 1, 2 and 4. (7 marks)

b) Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$

i. Find the characteristic polynomial of A. (2 marks)

ii. Find all the eigenvalues λ_1 and λ_2 of A and their corresponding eigenvectors. (4 marks)

iii. Is A diagonalizable? If yes, Determine the matrices P and D such that $D = P^{-1}AP$ such that D is diagonal. (2 marks)

iv. Consider a polynomial $f(x) = x^3 - 5x^2 + 3x + 6$. Find $f(\lambda_1)$ and $f(\lambda_2)$,hence find $f(A)$ (3 mark)

v. Find a matrix B such that $B^2 = A$ (2 marks)

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