CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS, ACTUARIAL SCIENCE

MATH 321: CALCULUS III

STREAMS: "AS ABOVE"

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 31/3/2021 **2.30 PM – 4.30 PM**

INSTRUCTIONS:

- Answer question **ONE** and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) Evaluate $\lim_{n\to\infty} \frac{3n}{5n-4}$ (3 marks)

b) Evaluate $\int_0^1 \int_{x^2}^{x} xy(x+y)dydx$ (5 marks)

c) Verify the mean value theorem (MVT) for $f(x) = x^3 - x^2 - x - 1$on the interval [0,2] (5 marks)

d) Evaluate the triple integral $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x+y+z)\, dydxdz$ (5 marks)

e) Determine whether or not the function $z = x^2y^3 + x^4y + xe^y$ is harmonic (5 marks)

f) Find the area of the region bounded by $x = \sqrt{y}$ (4 marks)

g) For an arbitrary function *z=f(x, y)* define the partial derivatives with respect to x and y (3 marks)

QUESTION TWO (20 MARKS)

a) Find and classify the critical points of the function $f(x) = x^4 + y^4 - 4xy + 2$ (8 marks)

b) Consider the integral $\iint_R xy^2 dA$,where R is defined by $0 \le x \le 2$ and $0 \le x \le 1$

Show that $\iint_R xy^2 dxdy = \iint_R xy^2 dydx$ (6 marks)

c) Find the Maclaurin's series for $f(x) = sin\pi x$ upto x^5 (6 marks)

QUESTION THREE (20 MARKS)

a) Find the average of $f(x,y) = xy$ on the region *R={[0,2]x[0,3]}* (4 marks)

b) Given that $f(x,y) = e^{xy} + ylnx$, find f_x and f_y and comment (4 marks)

i) State the Ratio test and hence

ii) Determine whether or not the series $\sum_{n=1}^{\infty} \frac{2^k k!k!}{(2k)!}$ Converge (4 marks)

c) Find the area between the parabola $y = x^3$ and $y = 2x + 3$ (4 marks)

d) Calculate the volume under the surface $z = 3 + x^2 - 2y$ over the region D defined by $0 \le x \le 1$ and $-x \le y \le x$ (4 mark)

QUESTION FOUR (20 MARKS)

a) Using power series of sin x and cos x show that $tanx \approx x + \frac{x^3}{3} + \frac{2x^5}{15}$ (5 marks)

b) Use double integral to find the area of $R = \{(x,y)\varepsilon R^2 : x\varepsilon[-1,2], y\varepsilon[x^2, x+2]\}$ (5 marks)

c) Find the mass, center of mass and moments of inertia of a plate on the triangle bound by $0 \le x \le 1$ and $0 \le y \le x$ whose density is $\ell(x,y) = y^2$ (10 marks)

QUESTION FIVE (20MRKS)

a) Evaluate the integral $\iint_R xy(x+y)dxdy$ over the area between the curves $y = x^2$ and $y = x$ (8 marks)

b) State the basic comparison test for series convergence and then determine the convergence of $\sum_{k=2}^{\infty} \frac{1}{n^2 + 1}$ (7 marks)

c) Find the relative maximum and minimum vales of $f(x,y) = x^2 + xy + y^2 - 3x$ (5 marks)

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