CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ENGINEERING

MATH 323: NUMERICAL ANALYSIS 1

STREAMS: **TIME: 2 HOURS**

DAY/DATE: FRIDAY 24/09/2021 **11.30 A.M – 1.30 P.M**

INSTRUCTIONS

Answer Question ONE and any other TWO Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a. Show that the equation $e^{-x} = x$ has a root in the interval $[0,1]$ (3marks

b. Solve the system using Gaussian Elimination (5marks)

$$2x + 3y - z = 5$$
$$4x + 4y - 3z = 3$$
$$2x - 3y + z = -1$$

c. Consider the table of values below

x	1	3	5
y	1.5708	1.5719	1.5738

Use Lagranges Interpolation to construct a linear polynomial and hence evaluate $f(3.5)$ (4marks)

d. The table of values x of and the corresponding y is given below (5marks)

x	1.50	1.65	1.80	1.95	2.05	2.25	2.40	2.55	2.70	2.85	3.00
y	1.025	1.081	1.132	1.182	1.249	1.308	1.375	1..438	1.538	1.571	1.623

Use Simpson's 1/3 rule to T given that $(0.018)T = \int_{1.5}^{3.0} y dx$ correct to 3 dp

e. Use row reduction to solve the system of equations (5marks)

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + y + 3z = 14$$

f. Use Simpson's 3/8 rule to evaluate $I = \int_0^{\pi/2} e^{Sin\,x}\,dx$ (4marks)

x	0	$\pi/6$	$\pi/3$	$\pi/2$
$y = e^{Sinx}$	1	1.64872	2.36320	2.71822

g. Use Newton forward interpolating formula to evaluate $f(15)$ given the data in the table below (4marks)

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

QUESTION TWO (20 MARKS)

a. Find the quadratic alagrange Interpolating polynomial for the data $x_0 = 2, x_1 = 2.5, x_2 = 4$ and $f(x) = \frac{1}{x}$. Hence approximate $f(3)$ (7marks)

b. Use the Secant method to find the root of the equation $x^3 - 4 = 0$ to 5dp with $x_1 = 1$ and $x_2 = 1.5$ (7marks)

c. For the data, determine $f(1895)$ using Newton's Forward difference formula (6marks)

x	1891	1901	1911	1921	1931
$f(x)$ *in million*	46	66	81	93	101

QUESTION THREE (20 MARKS)

a. Use the Trapezoidal rule with $n = 6$ to evaluate $\int_0^{0.5\pi} Sin\,x\,dx$ to 4 dp and find the error in the approximation (6marks)

b. Use the Bisection method to solve $x^3 - 9x + 1 = 0$ for the root in the interval $[2,4]$ after 5 iterations (6marks)

c. Find y' and y'' using the values in the table and the Newton backward difference formula (6marks)

x	1.4	1.6	1.8	2.0	2.2
$y=f(x)$	4.0552	4.9530	6.0496	7.3891	9.0250

QUESTION FOUR (20 MARKS)

a. State four advantages polynomials as interpolating functions (4marks)

b. Consider the table of data below

x	3.30	3.40	3.50	3.60
$y=f(x)$	0.303030	0.294118	0.285714	0.277778

Using Newton's divided differences, find the interpolating polynomial that fits the data and then approximate $f(3.55)$ (7marks)

c. Solve the system using matrix inversion method (5marks)

$$x+2y+3z=6$$
$$x+y+2z=1$$
$$x+3y+4z=6$$

d. Compute using Simpson's 1/3 rule the velocity of the missile when $t=80s$ using the data in the table below (4marks)

t	0	10	20	30	40	50	60	70	80
a=$\frac{dv}{dt}$	30.00	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

QUESTION FIVE (20 MARKS)

a. Use the Newton Raphson formula for finding the fourth root to evaluate $\sqrt[4]{29}$ to 4dp after 5 iteration (6marks)

b. Given that ,find the approximate relative error in at $x=y=z=1$ and $\delta x=\delta y=\delta z=0.001$ (6marks)

c. Use the Romberg method to find $\int_1^{1.8} y\,dx$ starting with Trapezoidal for the tabular values with

$h_1=0.4, h_1=0.2, h_1=0.1,$ (7marks)

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
y	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.823	3.107