

CHUKA UNIVERSITY

UNIVERSITY EXAMINATION

UNIVERSITY EXAMINATION FOR THE AWARD DEGREE OF BACHELOR OF SCIENCE GENERAL

MATH 323: NUMERICAL ANALYSIS I

DAY/DATE : JULY 2021

TIME: 2HOURS

INSTRUCTIONS:

Answer Questions ONE (compulsory) and any other TWO Questions

QUESTION ONE (30 MARKS) COMPULSORY

- a. Use the trapezoidal rule with $n = 4$ to estimate $\int_1^2 \frac{1}{x} dx$ and Compare the estimate with the exact value of the integral (6 marks)
- b. Find A^{-1} by **Gaussian** elimination on the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$ (5Marks)
- c. Using Lagranges interpolation formula find $y(10)$ from the table below (5Marks)

x	5	5	9	11
y	12	13	14	16

- d. Using Newton's Backwards Finite Difference Interpolation Formula, find $y(10)$ from the table below (5Marks)

x	5	5	9	11
y	12	13	14	16

- e. Solve the equation using the Secant method for 3 iterations with $x_0 = 4$ and $x_1 = 5.5$

$$x^3 - 20 = 0$$

(5Marks)

f. Evaluate the error, absolute error and percentage error in the number 3-1415926536 and

its approximate value 3.14159265

(4Marks)

QUESTION TWO (20 MARKS)

a. Use Newtons divided differences formula to evaluate $f(8)$ and $f(15)$ (7Marks)

x	4	5	7	10	11	13
$y = f(x)$	48	100	294	900	1210	2025

b. Compute $\int_0^1 \frac{dx}{1+x}$ correct to 4d.p for $h=0.5, \frac{h}{2} = 0.25, \frac{h}{4} = 0.125$ using **Romberg** method

(7Marks]

c. Find the second derivative of the function at $x=1.5$ based on the finite differences for the data

given in table below

(6 Marks)

X	1.5	2	2.5	3	3.5	4
F(x)	3.375	7	13.625	24	38.875	59

QUESTION THREE (20MARKS)

a. Using Newton Raphson Method, obtain $\sqrt[3]{12}$ to 4 decimal places with 5 iterations and $x_0 = 3$

(7Marks).

b. Solve the system of equations using the Gauss Elimination Method

(7Marks)

$$x + y + z = 7$$

$$3x + 3y + 4z = 24$$

$$2x + y + 3z = 16$$

c. Compute the integral using **Simpson's 1/3 rule** taking $h=0.125$

(6 marks)

$$I = \sqrt{\frac{2}{\pi}} \times \int_0^1 e^{-\frac{x^2}{2}} dx$$

QUESTION FOUR (20 MARKS)

- a. The table below gives the values of $\tan x$ for $0.10 \leq x \leq 0.30$

x	0.1	0.15	0.2	0.25	0.3
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

Find: (a) $\tan 0.12$ (b) $\tan 0.26$ using NFDIF and NBDIF (8Marks)

- b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using **Simpson's** $\frac{3}{8}$ th rule taking $n = 6$

[6 Marks]

- c. Solve the equation for a solution in the interval $(1.5, 2)$ using the Regula Falsi method with 5 iterations (6Marks)

QUESTION FIVE (20 MARKS)

- a. Set up Newton iteration for computing the square root of a positive number hence find the square root of 2 correct to six decimal places. (7 Marks)

- b. Using Newton's forward Interpolation formula find $f(8)$ from the table given. [7 Marks]

X	0	5	10	15	20	25
F(x)	7	11	14	18	24	32

- c. Use **Cramer's rule** to solve the system of linear equations given by $2x - 3y + 4z = 33$
 $3x - 2y - 2z = 2$

$$x + y + z = 4$$

(6Marks)

