

CHUKA

UNIVERSITY



UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE GENERAL

MATH 323: NUMERICAL ANALYSIS I

STREAMS:

TIME: 2 HOURS

DAY/DATE: WEDNESDAY 14/07/2021

8.30 A.M. – 10.30 A.M.

INSTRUCTIONS:

Answer Questions ONE (compulsory) and any other TWO Questions

QUESTION ONE (30 MARKS) COMPULSORY

- a. Use the trapezoidal rule with $n = 4$ to estimate $\int_1^2 \frac{1}{x} dx$. and Compare the estimate with the exact value of the integral (6 marks)
- b. Find A^{-1} by **Gaussian** elimination on the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$ (5 Marks)
- c. Using Lagranges interpolation formula find $y(10)$ from the table below (5Marks)

x	1	3	4	5	7	10
y	3	31	69	131	351	1011

- d. Using Newton's Divided Finite Difference Interpolation Formula, find $y(8)$ from the table below (5 Marks)

x	5	5	9	11
y	12	13	14	16

- e. Solve the equation using the Secant method for 3 iterations with $x_0 = 4$ and $x_1 = 5.5$

$$x^3 - 20 = 0 \quad (5 \text{ Marks})$$

- f. Evaluate the error, absolute error and percentage error in the number 3.1415926536 and its approximate value 3.14159265 (4 Marks)

QUESTION TWO (20 MARKS)

- a. Use Newton's divided differences formula to evaluate $f(8)$ and $f(15)$ (7 Marks)

x	4	5	7	10	11	13
$y = f(x)$	48	100	294	900	1210	2025

- b. Compute $\int_0^1 \frac{dx}{1+x}$ correct to 4d.p for $h=0.5, \frac{h}{2} = 0.25, \frac{h}{4} = 0.125$ using **Romberg** method (7 Marks]
- c. Find the second derivative of the function at $x=1.5$ based on the finite differences for the data given in table below (6 Marks)

X	1.5	2	2.5	3	3.5	4
F(x)	3.375	7	13.625	24	38.875	59

QUESTION THREE (20 MARKS)

- a. Using Newton Raphson Method, obtain $\sqrt[3]{12}$ to 4 decimal places with 5 iterations and $x_0 = 3$

(7 Marks)

- b. Solve the system of equations using the Gauss Elimination Method

(7 Marks)

$$\begin{aligned} x + y + z &= 7 \\ 3x + 3y + 4z &= 24 \\ 2x + y + 3z &= 16 \end{aligned}$$

- c. Compute the integral using **Simpson's 1/3 rule** taking $h=0.125$

(6 marks)

$$I = \sqrt{\frac{2}{\pi}} \times \int_0^1 e^{-\frac{x^2}{2}} dx$$

QUESTION FOUR (20 MARKS)

- a. The table below gives the values of $\tan x$ for $0.10 \leq x \leq 0.30$

x	0.1	0.15	0.2	0.25	0.3
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

Find: (a) $\tan 0.12$ (b) $\tan 0.26$ using NFDIF and NBDIF

(8 Marks)

- b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using **Simpson's $\frac{3}{8}$ th rule** taking $n = 6$

[6 Marks]

- c. Solve the equation for a solution in the interval $(1.5, 2)$ using the Regula Falsi method with 5 iterations

(6 Marks)

QUESTION FIVE (20 MARKS)

- a. Set up Newton iteration for computing the square root of a positive number hence find the square root of 2 correct to six decimal places.

(7 Marks)

- b. Using Newton's forward Interpolation formula find $f(8)$ from the table given. [7 Marks]

X	0	5	10	15	20	25
F(x)	7	11	14	18	24	32

- c. Use **Cramer's rule** to solve the system of linear equations given by
- $$\begin{aligned}x + y + z &= 4 \\ 2x - 3y + 4z &= 33 \\ 3x - 2y - 2z &= 2\end{aligned}$$

(6 Marks)

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