CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 341: SAMPLING METHODS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: MONDAY 05/07/2021 **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:

QUESTION 1 (30 MARKS)

(a) Define the following terms as used in sampling methods and give an example of each.

(i) A census and a sample survey (2 marks)

(ii) Simple random sampling (2 marks)

(iii) Stratified sampling (2 marks)

(b) (i) State two advantages and two disadvantages of a sample survey over a census. (4 marks)

(ii) State four advantages of stratified sampling. (2 marks)

(c) In a population of N=6 the values y_i are 29, 32, 27, 30, 34, 28. By enumerating all possible sample of size 5.

(i) Show that the relation $E(\bar{y}) = \bar{y}$ is satisfied. (5 marks)

(ii) Show that $E(S^2) = \sigma^2$ (5 marks)

(d) (i) State two advantages of systematic sampling. (2 marks)

(ii) In systematic sampling, suppose K=8 and the first unit drawn is 42. If N=65 determine the three subsequent units. (3 marks)

(e) (i) Show that the estimator $\hat{Y} = N\bar{y}$ is unbiased estimator of the population. (2 marks)

(ii) Show that the stratified mean $\bar{y}_{st}$ is an unbiased estimator of the stratum mean. (3 marks)

QUESTION 2 (20 MARKS)

(a) The following table gives the results of a stratified random sample drawn from a population of 41.

Stratum (n)	$N_{(n)}$	n_n	$\bar{y}_n$	$s^2 n$
1	20	3	1.6	3.3
2	10	5	1.8	4.0
3	11	4	0.6	2.2

Determine (a) $\bar{y}_{st}$ (2 marks)

(b) var $(\bar{y}_{st})$ (3 marks)

(c) $\hat{y}$, the estimate of the population total. (2 marks)

(b) Prove that the variance of the systematic mean is given by

$$V(\bar{y}_{sy} = \left(\frac{N-1}{N}\right) S^2 - \frac{K(n-1)}{N}\, S^2_{wsy}$$

Where $\sigma^2 = \frac{1}{N-1}\sum_{j=1}^{n}\sum_{i=1}^{k}(y_{ij} - \bar{y})^2$ is the population mean and

$\sum^2_{wsy} = \frac{1}{k(n-1)}\sum_{j=1}^{n}\sum_{i=1}^{k}(y_{ij} - \bar{y})^2$ is the variance of the units within the same sample. (13 marks)

QUESTION 3 (20 MARKS)

(a) A simple random sample of $n = 20$ pages out of the N=185 pages in the book pilgrims progress by John Bunyan gave the following number of words per page.

298, 363, 340, 265, 302, 354, 317, 385, 361, 226, 270, 245, 323, 275, 211, 119,281, 225, 331, 304

(i) Estimate the mean number of words per page and its standard error. (7 marks)

(ii) Construct a 95% confidence interval for the estimate of the population mean. (3 marks)

(iii) Estimate the total numbers of words in the book and its standard error. (3 marks)

(b) Show that in simple random sampling without replacement

$E(s^2) = s^2$ where $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ and $\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$ (7 marks)

QUESTION 4 (20 MARKS)

Prove that var $(\bar{y}_{st}) = \frac{N-1}{N}\frac{S^2}{n}(1 + (n-1)p_w)$ where p_w is the correlation coefficient between pairs of units that lie in the same systematic sample given by

$p_w = \frac{E(y_{ij}-\bar{y})(y_{in}-\overline{\bar{y}})}{E(\bar{y}_{ij}-\overline{\bar{y}})^2}$ (20 marks)

QUESTION 5 (20 MARKS)

(a) Suppose in a stratified random sampling the cost function is of the form

$C = C_0 + \sum_{n=1}^{i} C_n\, n_n$

Show that the optimum sample size of the n^{th} stratum is given by

$\cap_n = \frac{nN_n s_n}{\sum_{n=1}^{l} w_n s_n}$ (7 marks)

(b) State three rules which would lead one to take a larger sample size for a given stratum. (3 marks)

(c) A sampler proposes to take a stratified sample. He expects that his field costs will be of the form

$\sum_{n=1}^{L} C_n \cap_n$. His estimates of the relative quantities are as follows.

Stratum	w_n	s_n	c_n
1	0.4	10	Ksh. 400
2	0.6	20	Ksh.900

(i) Find the values of $\frac{n_1}{n}$ and $\frac{n^2}{n}$ that minimize the total field cost for a given value of var $(\bar{y}_{st})$ (4 marks)

(ii) Find the sample size required under the optimum allocation to make var $\bar{y}_{st} = 1$. Ignore the finite population correlation factor (Assure N is large). (2 marks)

(iii) Determine the total field cost. (4 marks)
