CHUKA UNIVERSITY

JAN-MARCH 2021 EXAMINATION

MATH 344: THEORY OF ESTIMATION

STREAMS: BSc. ECONSTAT, BA ECON MATH, BED Sc.,

TIME 2 HOURS

Instructions

- Answer Question ONE and any other TWO questions.
- All workings must be shown clearly

QUESTION 1[30 MARKS]

a) Define the following terms as used in theory of Estimation
   (i) Mean square error consistency
   (ii) An estimator
   (iii) Unbiasedness
   (iv) Sufficient statistic [8 marks]

b) Which one of the following is not an unbiased estimator of θ, given that $E(x_i) = \theta$
   $T_1 = x_1 + x_2 + x_3 + x_4$
   $T_2 = 2x_1 + 3x_2$
   $T_3 = 4x_2 - 3x_3$ [3 marks]

c) Differentiate between Point and Interval estimation [4 marks]

d) Let $x_i, i = 1,2,3,4$, be four independent sample observations of Poisson distribution with parameter θ. Show that $T = \frac{1}{15}(2x_1 + 4x_2 + 5x_3 + 3x_4)$ is a biased estimator of θ. Calculate the amount of bias. [5 marks]

e) Let T_1 be the most efficient estimator and T_2 be the unbiased estimator for unknown parameter θ. If ρ is the efficiency with respect to T_1, show that
   $Var(T_1 - T_2) = \frac{1-\rho}{\rho}\, var(T_1)$ [5 marks]

f) Find the sample size (n) of a random sample taken from a normal population with mean μ and variance 9 and given that $\bar{X}$ is the mean of the random sample such that
   $P[\bar{x} - 1 < \mu < \bar{x} + 1] = 0.9$ [5 marks]

QUESTION 2 [20 MARKS]

a) Consider two random samples $x_1, x_2, \dots, x_{n1}$ of size n1 and $y_1, y_2, \dots, y_{n2}$ of size n2 both from normal populations such that $x \sim N(\mu_1, \sigma_1^2)$ and $y \sim N(\mu_2, \sigma_2^2)$ respectively. Obtain the $(1-\alpha)100\%$ confidence interval for $(\bar{x} - \bar{y})$. [7 marks]

b) The distribution of $x$is given by $f(x) = \begin{cases} \theta^x(1-\theta)^{1-x} & ,x = 0,1 \\ 0 & elsewhere \end{cases}$

Show that $T = \sum x_i$ is a sufficient statistic for θ. [8Marks]

QUESTION 3 [20 MARKS]

a) Given $f(x,\theta) = \frac{1}{\pi}.\frac{1}{1+(x-\theta)^2}, -\infty < x < \infty$

Find $I(\theta)$ [15 Marks]

c) Find sufficient statistic for δ^2 where $x \sim N(\mu, \delta^2)$ [5 Marks]

QUESTION 4 [20 MARKS]

a) Define a uniformly minimum variance unbiased estimator (UMVUE) T of $\tau(\theta)$. [5 Marks]

b) If T is a consistent estimator of θ, $\emptyset(T)$ is also a consistent estimator of $\emptyset(\theta)$ where $\emptyset$ is a continuous function, Proof. [15 Marks]

QUESTION 5 [20 MARKS]

a) Let $x_1, x_2 \ldots x_3$ be a random sample from a Poisson distribution with parameter θ. Using the Cramer-Rao inequality condition, show that the mean $\bar{x}$ is UMVUE of the population mean. [12 Marks]

b) Let $y_1, y_2, \ldots y_n$ be a random sample with a distribution given as

$$f(x) = \begin{cases} \frac{2(\beta - y)}{\beta^2}, 0 < y < \beta \\ 0, \quad otherwise \end{cases}$$

i) Find an estimator of β by method of moments [4 Marks]

ii) Determine if the estimator is unbiased [4 Marks]