CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 344 : THEORY OF ESTIMATION

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: MONDAY 01/11/2021 **11.30 A.M. – 1.30 P.M.**

INSTRUCTIONS:

- **Answer all questions.**
- **All workings must be shown clearly.**

QUESTION 1[30 MARKS]

a) Define the following terms as used in theory of Estimation

(i) Mean square error consistency

(ii) An estimator

(iii) Unbiasedness

(iv) Efficient statistic [8 marks]

b) Which one of the following is not an unbiased estimator of θ,

given that $E(x_i) = \theta$

$T_1 = x_1 + x_2 + x_3 + x_4$

$T_2 = 2x_1 + 3x_2$

$T_3 = 4x_2 - 3x_3$ [3 marks]

c) Differentiate between Point and Interval estimation [4 marks]

d) Let $x_i, i = 1,2,3,4$, be four independent sample observations of Poisson distribution with parameter θ. Show that $T = \frac{1}{15}(2x_1 + 4x_2 + 5x_3 + 3x_4)$ is a biased estimator of θ. Calculate the amount of bias. [5 marks]

e) Let T_1 be the most efficient estimator and T_2 be the unbiased estimator for unknown parameter θ. If ρ is the efficiency with respect to T_1, show that

$Var(T_1 - T_2) = \frac{1-\rho}{\rho}\ var(T_1)$ [5 marks]

f) Find sufficient statistic for δ^2 where $x \sim N(\mu, \delta^2)$ [5 marks]

QUESTION 2 [20 MARKS]

a) Consider two random samples $x_1, x_2, \ldots, x_{n1}$ of size n1 and $y_1, y_2, \ldots, y_{n2}$ of size n2 both from normal populations such that $x \sim N(\mu_1, \sigma_1^2)$ and $y \sim N(\mu_2, \sigma_2^2)$ respectively. Obtain the $(1-\alpha)100\%$ confidence interval for $(\mu_1 - \mu_2)$. [7 marks]

b) The distribution of $x$is given by $f(x) = \begin{cases} \theta^x (1-\theta)^{1-x} & , x = 0,1 \\ 0 & elsewhere \end{cases}$

Show that $T = \sum x_i$ is a sufficient statistic for θ. [8 Marks]

QUESTION 3 [20 MARKS]

a) Define a uniformly minimum variance unbiased estimator (UMVUE) T of $\tau(\theta)$. [5 Marks]

b) If T is a consistent estimator of θ, $\emptyset(T)$ is also a consistent estimator of $\emptyset(\theta)$ where $\emptyset$ is a continuous function, Proof. [15 Marks]
