THIRD YEAR **SECOND** SEMESTER EXAMINATION FOR DEGREE OF
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
MATH 347 – PROBABILITY MODELLING

DURATION: 2 HOURS
DATE:
TIME:

Instructions to Candidates:

1. Answer **Question 1** and **Any Other Two** questions.
2. Mobile phones are not allowed in the examination room.
3. You are not allowed to write on this examination question paper.

SECTION A – ANSWER ALL QUESTIONS IN THIS SECTION

QUESTION ONE

a)Define the following

i)counting process (2 marks)

ii)stochastic process (2 marks)

iii)Bernoulli process (2 marks)

b) In a discrete-time Markov chain, there are two states 0 and 1. When the system is in state 0 it stays in that state with probability 0.4. When the system is in state 1 it transitions to state 0 with probability 0.8 . Graph the Markov chain and find the state transition matrix P.(4 marks)

c) Suppose that 40% of the voters in a city are in favor of a ban on smoking in public buildings. Suppose 5 voters are to be randomly sampled. Find the probability that

i)2 favor the ban. (2 marks)
ii)less than 4 favor the ban. (2 marks)
iii)at least 1 favor the ban. (2 marks)

d) Consider the following single-server queue: the inter-arrival time is exponentially distributed with a mean of 10 minutes and the service time is also exponentially distributed with a mean of 8 minutes, find the

(i) mean wait in the queue (2 marks)

(ii) mean number in the queue (2 marks)

(iii) the mean wait in the system (2 marks)

(iv) mean number in the system and (2 marks)

(v)proportion of time the server is idle (2 marks)

e)List and briefly explain any two types of stochastic processes.(4 marks)

SECTION B – ANSWER ANY TWO QUESTIONS IN THIS SECTION

QUESTION TWO

a)Distinguish between strict stationarity and stationarity in the weak sense.(4 marks)

b) Consider a Markov process with state space **S** ={0,1, 2} and transition matrix, **P**:

$$P = \begin{pmatrix} p & q & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ p-\frac{1}{2} & \frac{7}{10} & \frac{1}{5} \end{pmatrix}$$

i) What can you say about the values of **p** and **q** ?(3 marks)
ii) Calculate the transition probabilities $\mathbf{p_{ij}}^{(3)}$(5 marks)
iii) Draw the transition graph for the process represented by **P (4 marks).**

c)Explain the equivalence property of a queueing system.(4 marks)

QUESTION THREE

a) What is queue discipline?(2 marks)

b) Is the process with the following transition matrix irreducible?(3 marks)

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

c) What are the stationary distributions of the process above?(7 marks)

d) Discuss what the following states mean in markov chain .(6 marks)
i) transient
ii) recurrent
iv) absorbing

e) When is a state i said to be periodic.(2 marks)

QUESTION FOUR

a)Babies are born in Chuka at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following

I. The average number of births per year(2 marks)
II. The probability that no births will occur in one day(3 marks)
III. The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3-hour period(3 marks)

b) The matrix P specified below is the transition matrix of a Markov chain.

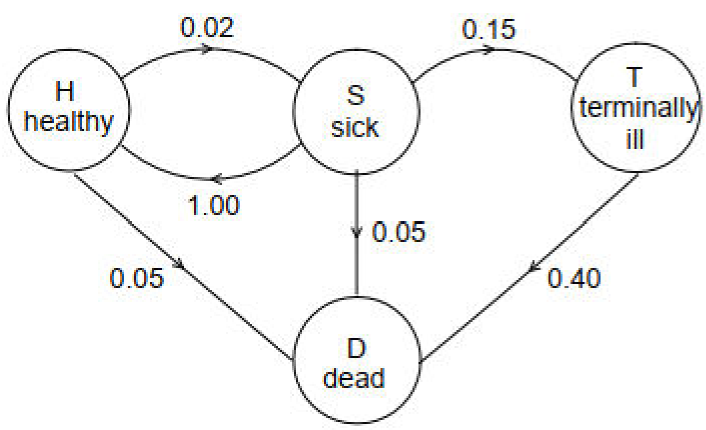
$$P = \begin{pmatrix} & 1 & 2 & 3 & 4 & 5 \\ 1 & 2/3 & 1/3 & 0 & 0 & 0 \\ 2 & 2/3 & 0 & 1/3 & 0 & 0 \\ 3 & 0 & 2/3 & 0 & 1/3 & 0 \\ 4 & 0 & 0 & 2/3 & 0 & 1/3 \\ 5 & 0 & 0 & 0 & 2/3 & 1/3 \end{pmatrix}$$

(a) Which states are transient and which, recurrent? (2 marks)
(b) What is the period of state 2?(2 marks)
(c) What is the probability of returning to state 2 if we start from 2? (2 marks)
(d) What is the probability of reaching state 1 from state 2? (2 marks)
(e) What is its stationary distribution? (3 marks)
(f) Let N2 denote the number of times the chain returns to 2 in time n: What is

$\lim_{n \to \infty} E(N2)/n$ (1mark)

QUESTION FIVE

a) Consider the following Health, Sickness, Death model with the addition of an extra "Terminally ill" state, T. The rates given are per year.



i) Calculate the expected holding time in state S (2 marks)
ii) Calculate the probability that a sick life goes into state D when he leaves the sick state.(2 marks)

iii) Calculate the expected future lifetime of a healthy life.(6 marks)

b) The credit-worthiness of debt issued by companies is assessed at the end of each year by a credit rating agency. The ratings are A (the most credit-worthy), B and D (debt defaulted). Historic evidence supports the view that the credit rating of a debt can be modelled as a Markov chain with one-year transition matrix:

$$\mathbf{X} = \begin{pmatrix} 0.92 & 0.05 & 0.03 \\ 0.05 & 0.85 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

i) Determine the probability that a company currently rated A will never be rated B in the future(2 mark)

ii)Calculate the second order transition probabilities of the Markov chain(2 marks)

iii)Hence calculate the expected number of defaults within the next two years from a group of 100 companies, all initially rated A(2 marks)

The manager of a portfolio investing in company debt follows a "downgrade trigger" strategy. Under this strategy, any debt in a company whose rating has fallen to B at the end of a year is sold and replaced with debt in an A-rated company.

iv)Calculate the expected number of defaults for this investment manager over the next two years, given that the portfolio initially consists of 100 A-rated bonds(2 marks)

v) Comment on the suggestion that the downgrade trigger strategy will improve the return on the portfolio(2 marks)