CHUKA UNIVERSITY

SUPPLEMENTARY/ SPECIAL EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF

MATH 400/401: TOPOLOGY I

STREAMS: **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY03/02/2021 **8.30 AM -10.30 AM**

INSTRUCTIONS:

- Answer question **ALL** the questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Distinguish the following terms as used in topology

(i) An indiscrete topology and a discrete topology

(ii) A dense subset and a nowhere dense subset

(iii) The interior of the point p and a neighborhood of the point p

(iv) A base and a subbase for the topology τ

(v) $a\, T_1$ $and\, T_2$ space (10mks)

(b) Let $p \in X$ and denote $N_P$the set of all neighborhood of a point p. Prove that if $N \in N_P$ and for every $M \subset X$ with $N \subset M$ it implies that $M \in N_P$ (3mks)

(c) Show that if X be a discrete topological space and that $A \subset X$, then the derived set of A, $A^{'} = \emptyset$. (4mks)

(d) Let $f: x_1 \rightarrow x_2$ $where\, x_1 = x_2 = \{0,1\}$ and are such that $(x_1, D) and$ $(x_2, \$)$ be defined by $f(1) = 1$ $and\, f(0) = 0$. Show that f is not continous but f^{-1} is continuous. (4mks)

(e) Define the term a topological property (p). Give an example for this property. (2mks)

(f) Distinguish between a $a\, T_1$ $and\ T_2$ space. Using an appropriate counter example show that a T_2 space $\Rightarrow T_1$space but a T_1 space $\nRightarrow T_2$. (7mks)

QUESTION TWO: (20 MARKS)

(a) (i) Define a limit point of a subset A of a topological space X. (1mk)

(ii) Let $X = \{a, b, c, d, f\}$ and $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{c\}, \{a, f\}, \{f\}, \{b, c, f\}, \{a, b, c, f\}\}$. Let $A = \{a, c, d\}$. Show that b is a limit point of A but a is not. (6mks)

(b) Let (X, τ) be a topological space and $A, B \subset X$. Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$ (4mks)

(c) Let (X, τ) be a topological space and $A, B \subset X$. Denote A^0 the interior of A.

(i) Using an appropriate example, show that $A^0 \cup B^0 \neq (A \cup B)^0$ (4mks)

(ii) Prove that $A^0 \cap B^0 = (A \cap B)^0$ (5mks)

QUESTION THREE: (20 MARKS)

(a) Consider the following topology on $X = \{a, b, c, d, e\}$ and $\tau = \{\{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}, X, \emptyset\}$. If $A = \{a, b, c\}$. Find

(i) The exterior of A (3mks)

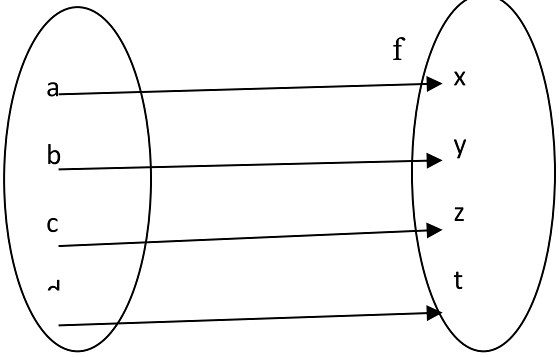
(ii) The boundary of A (3mks)

(iii) Hence show that the boundary of A, $\delta A = \bar{A} \cap \overline{X/A}$ (3mks)

(b) Let $X = \{a, b, c, d\}$ $with\ \tau_X = \{\{a, b\}, \{a\}, \{b\}, X, \emptyset\}$ and Let $Y = \{x, y, z, t\}$ $with$

$\tau_Y = \{\{x\}, \{y\}, \{x, y\}, Y, \emptyset\}$.

Define the function f as



Show that the function f is a homomorphism. (5mks)

(c) Prove that every metric space is a T_2 space. (6mks)
