CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN APPLIED COMPUTER SCIENCE

MATH 407: FOURIER ANALYSIS

STREAMS: BSC ACSC **TIME: 2 HOURS**

DAY/DATE: MONDAY 22/03/2021 **8.30 A.M. – 10.30 A.M.**

INSTRUCTIONS

- **Answer question one and any other two questions.**
- **Adhere to the instructions on the answer booklet.**

QUESTION ONE (COMPULSORY)

a. Find the complex Fourier series for $f(x)=\begin{cases}0, -\pi \le x \le 0\\ 2, 0 \le x \le \pi\end{cases}$ and show that $c_n = \begin{cases}\frac{1}{in\pi}, n \text{ is odd}\\ 0, n \text{ is even}\end{cases}$

(5 marks)

b. Evaluate $\int_0^1 \left(1-x^3\right)^{-\frac{1}{2}} dx$ using the Beta function (5 marks)

c. Using the Fourier cosine integral representation of an appropriate function, show that

$\int_0^\infty \frac{\cos wx}{k^2+w^2} dw = \frac{\pi e^{-kx}}{2k}$ (5 marks)

d. Obtain the Fourier sine transform of $f(x) = \frac{e^{-3x}}{x}$ (5 marks)

e. Determine the exponential form of the Fourier series for the function defined by $f(t) = 2t,\ -\pi \le t \le \pi$

(5 marks)

f. Evaluate $\int_0^\infty x^{n-1}e^{-4x^2}\,dx$ by the Gamma function (5 marks)

QUESTION TWO

a. State the Dirichlets conditions for a Fourier series (5 marks)

b. Find the Fourier series representing $f(x)=x, 0<x<2\pi$ (7 marks)

c. Obtain a_0, a_n *and* b_n for the periodic function $f(x)=\begin{cases}0, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$ (8 marks)

QUESTION THREE

a. A periodic function of period 4 is defined as $f(x)=\begin{cases}x, & 0<x<2\\ -x, & -2<x<0\end{cases}$

Obtain a_0, a_n *and* b_n (8 marks)

b. Use the Fourier sine series for $f(x)=1$, in $0<x<\pi$ to show that $\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}......$

(7 marks)

c. Evaluate $\int_0^\infty \sqrt[4]{x}e^{-\sqrt{x}}\,dx$ (5 marks)

QUESTION FOUR

a. Find the Fourier cosine transform of

$$f(x)=e^{-2x}+4e^{-3x}$$

(6 marks)

b. Solve for f(x) from the integral equation $\int_0^\infty f(x)\cos sx\,dx=e^{-s}$ (7 marks)

c. Use Parsaval`s identity to show that $\int_0^\infty \frac{dx}{\left(x^2+1\right)^2}dt=\frac{\pi}{4}$ (7 marks)

QUESTION FIVE

a. Solve $U_t = kU_{xx} \quad for \;\; x \geq 0, t \geq 0$, under the given conditions $U = U_0 \; at \;\; x = 0, t > 0$, with initial conditions $U(x,0) = 0, x \geq 0$ by Fourier transforms. (8 marks)

b. Find the finite Fourier sine and cosine transform of $f(x) = x, \;\; in \;\; (0, l)$ (6 marks)

c. Find the finite Fourier sine transform of $f(x) = 1 \;\; in \;\; (0, \pi)$ and use the inversion theorem to prove that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$$ (6 marks)
