CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF BACHELOR DEGREE IN MATHEMATICS AND BACHELOR OF EDUCATION SCIENCE

MATH 407: FOURIER ANALYSIS

STREAMS: Bsc. MATHS & B.ED SCI.

TIME: 2 HOURS

DAY/DATE : TUESDAY 28 /09/ 2021 **11.30 AM – 1.30 PM**

INSTRUCTIONS

- **Answer question one and any other two questions**
- **Adhere to the instructions on the answer booklet.**

QUESTION ONE Compulsory.

a. State the Dirichlets conditions for a Fourier series [5 marks]

b. Evaluate $\int_0^1 \left(1-x^3\right)^{-\frac{1}{2}} dx$ using the Beta function [5 marks]

c. Given the function $f(x)=\begin{cases} 0, -\pi \le x \le 0 \\ 2, 0 \le x \le \pi \end{cases}$, Obtain c_n, the complex Fourier constant [5 marks]

d. Use Parsaval`s identity to show that $\int_0^\infty \frac{dx}{\left(x^2+1\right)^2} dt = \frac{\pi}{4}$ [6 marks]

e. Obtain the Fourier series for the derivative of $f(t)=t^2,\ (-\pi \le t \le \pi)$ [4 marks]

f. Obtain a_0, a_n *and* b_n for the periodic function $f(x)=\begin{cases}0, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$ [5 marks]

QUESTION TWO

a. Obtain the Fourier series for the integral of the function $f(t)=3t^2-\pi^2,\ (-\pi \le t \le \pi)$ [5 marks]

b. Find the Fourier cosine transform of

$$f(x)=e^{-2x}+4e^{-3x}$$

[6 marks]

c. Use the Fourier sine series for $f(x)=1$, in $0<x<\pi$ to show that $\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}$......
` [9marks]

QUESTION THREE

a. Using the Fourier cosine integral representation of an appropriate function, show that

$$\int_0^{\infty}\frac{\cos wx}{k^2+w^2}\,dw=\frac{\pi e^{-kx}}{2k}$$

[5 marks]

b. Determine the exponential form of the Fourier series for the function defined by $f(t)=2t,\ -\pi \le t \le \pi$ and find c_1 *to* c_5 [9 marks]

c. Evaluate $\int_0^{\infty}\sqrt[4]{x}\,e^{-\sqrt{x}}\,dx$ by the gamma function [6 marks]

QUESTION FOUR

a. Solve for f(x) from the integral equation $\int_0^{\infty} f(x)\cos sx\,dx=e^{-s}$ [7marks]

b. Find the Fourier sine integral for

$$f(x)=e^{-\beta x} \qquad (\beta>0)$$

hence show that $\frac{\pi}{2}e^{-\beta x}=\int_0^{\infty}\frac{\lambda \sin \lambda x}{\beta^2+\lambda^2}\,d\lambda$

[7 marks]

c. Find the function whose sine transform is

$$\frac{e^{-as}}{s}.$$

[7 marks]

QUESTION FIVE

a. Solve the heat transfer equation below by Fourier transforms
8mks

Solve $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ *for* $0 \le x < \infty,\ t > 0$ *given the conditions*

(i) $u\ (x,\ 0) = 0$ *for* $x \ge 0$ *(ii)* $\frac{\partial u}{\partial x}(0, t) = -a$ *(constant)*

(iii) $u(x,\ t)$ *is bounded.*

b. Solve $U_t = kU_{xx}$ *for* $x \ge 0,\ t \ge 0$, under the given conditions $U = U_0$ *at* $x = 0, t > 0$, with initial conditions $U(x,0) = 0, x \ge 0$ by Fourier transforms. [7 marks]

c. Evaluate $\int_0^\infty x^{n-1} e^{-4x^2} dx$ by the Gamma function [5marks]

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