CHUKA **UNIVERSITY**

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF SCIENCE MATHEMATICS

MATH 411: DIFFERENTIAL GEOMETRY

STREAMS: BSC. MATH **TIME: 2 HOURS**

DAY/DATE: TUESDAY 21/09/2021 **2.30 P.M. – 4.30 P.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) i) Define a regular representation function on an interval I. Hence show that the function $= (3t+1)\boldsymbol{e_1} + (t^4+5)\boldsymbol{e_2}\ , -\infty < t < \infty$ is a regular parametric representation.

(3 marks)

(ii) When is a real valued function $t = t(\theta)$ on an interval I_θ said to have an allowable change of parameter? Take $t = (b-a)\theta + a,\ 0 \le \theta \le 1\ , a < b$ to illustrate this.

(4 Marks)

b) Find the equation of the oscillating plane of the helix $\boldsymbol{x} = (cost)\boldsymbol{e_1} + (sint)\boldsymbol{e_2} + t\boldsymbol{e_3}$ at $\boldsymbol{t} = \frac{\pi^c}{2}$ (6 marks)

c) Compute the arc length to the curve $\boldsymbol{x} = 3(\cosh 2\,t)\boldsymbol{e_1} + \boldsymbol{3}(\sinh 2\,t)\boldsymbol{e_2} + 6t\boldsymbol{e_3}$ $0 \le t \le \pi$ (3 Marks)

d) Along the helix $= (acost)e_1 + (asint)e_2 + bte_3, a > 0, b \neq 0$. Find

(i) the unit tangent vector **t**. (3 marks)

(ii) the curvature vector **k** (3 marks)

(iii) the radius of curvature (2 marks)

e) (i) State without proof the Fundamental existence and uniqueness Theorem (2marks)

(ii) Show that the First Fundamental form is positive definite. (3 marks)

QUESTION TWO (20 Marks)

a) Define torsion $\tau(s)$ of the curve C at the point **x**(s). Hence show that the sign of τ is independent of the sense of principal normal vector **n** and the orientation of C. (6 marks)

b) State and prove the Serret-Frenet equations of a curve (8 marks)

c) Find the equations of the tangent line and normal plane to the curve

$\boldsymbol{x} = t\boldsymbol{e_1} + t^2\boldsymbol{e_2} + t^3\boldsymbol{e_3}$ at $t = 1$ (6 marks)

QUESTION THREE (20 Marks)

a) Find the equations of the tangent line and normal plane to the curve

$\boldsymbol{x} = t\boldsymbol{e_1} + t^2\boldsymbol{e_2} + t^3\boldsymbol{e_3}$ at $t = 1$ (6 marks)

b) Find the curvature and torsion of the curve $\boldsymbol{x} = (3t - t^3)\boldsymbol{e_1} + 3t^2\boldsymbol{e_2} + (3t + t^3)\boldsymbol{e_3}$ and compare your results. (7 marks)

c) When do we say an arc $\boldsymbol{x} = \boldsymbol{x}(t)$, $a \leq t \leq b$ said to be rectifiable? Hence or otherwise show that the arc $\boldsymbol{x} = t\boldsymbol{e_1} + t^2\boldsymbol{e_2}$, $0 \leq t \leq 1$ is rectifiable. (7 marks)

QUESTION FOUR (20 Marks)

a) i)Derive the First Fundamental form I to the coordinate patch $x = x(u, v)$ on a surface of class ≥ 2. (7 marks)

ii) Hence prove that the First Fundamental form depends only on the surface and not on the particular representation (7 marks)

b) Consider the surface represented by $\boldsymbol{x} = u\boldsymbol{e_1} + v\boldsymbol{e_2} + (u^2 - v^2)\boldsymbol{e_3}$. Find its second fundamental form II (6 marks)

QUESTION FIVE (20 Marks)

a) Prove that along a regular curve $\boldsymbol{x} = \boldsymbol{x}(s), the\ curvature\ \ |\boldsymbol{k}| = \frac{|x' \times x''|}{|x'|^3}$ (7 marks)

b) Distinguish an involute and an evolute of a curve C. Hence show that the curvature of an involute $\boldsymbol{x}^* = \boldsymbol{x} + (c - s)t$ of $= \boldsymbol{x}(s)$] is given by $k^{*^2} = \frac{k^2+\tau}{(c-s)^2k^2}$

(13 marks)

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