CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE AWARD OF BACHELOR OF EDUCATION SCIENCE [COMPUTER SCIENCE AND MATHEMATICS OPTION]

COSC 102: DISCRETE MATHEMATICS SOLVING

STREAMS: BSC. COMP. SCI (Y1S2) TIME: 2 HOURS

DAY/DATE: WEDENESDAY 31/03/2021 8.30 A.M. – 10.30 A.M

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** questions.
- Diagrams should be used whenever they are relevant to support an answer.
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely
- Electronic, non-programmable calculators may be used

SECTION A: COMPULSORY

QUESTION 1[30 Marks]

a) What is the definition of a *proposition*?, give an example [2 marks]

b) Which of the following sentences are propositions? Are they true or false?

i. $x + y = y + x$. [2 marks]

ii. $x^2 + 3 = 7$ [2 marks]

iii. For every positive real number *x*, $x + \frac{1}{x} \geq 2$ [2 marks]

iv. Every prime number is odd. [2 marks]

c) Write the *negation* of each of the following propositions without using any form of the word "not". For example, the negation of "She gets up before noon," is "She gets up at noon or later."

i. Today is Monday. [2 marks]

ii. 3 + 4 = 10 [2 marks]

iii. The weather in Chuka is cold and dry. [2 marks]

d) Use a *truth table* to determine whether the followings are a tautology, a contradiction, or a contingency.

i. $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ [2 marks]

ii. $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow \neg r$ [2 marks]

e) Show that the following pairs of propositions are *logically equivalent* **without** using a truth table.

i. $\neg(p \wedge (\neg p \vee q))$ and $\neg p \vee \neg q$ [2 marks]

ii. $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $p \leftrightarrow q$ [2 marks]

f) State the Converse, the inverse and the Contrapositive of the following conditional statement:
" The home team wins, wherever it is raining" [6 marks]

SECTION B: ATTEMPT ONLY TWO QUESTIONS FROM THIS SECTION
Question 2 [20 marks]

(a) Find the number of permutations of six objects, {A,B,C,D,E,F} taking three at a time [4 marks]

(b) Prove by direct proof or otherwise, that the sum of two odd numbers is even [4 marks]

(c) A farmer buys 3 cows, 2 pigs and 4 hens from a man who has 6cows, 5pigs, and 8 hens. Find the number of choices the farmer has to make

[12 marks]

Question 3[20 marks]

(a) Let M, P and C be the sets of students taking Mathematics, Physics and Computer courses respectively in Chuka University. Take |M| = 300, |P| =350, |C| = 450, |M∩P| = 100, |M∩C| = 150, and |P∩C|=75, |M∩N∩P∩C| =10. Determine the number of students taking exactly one of the above courses. [6 marks]

(b) How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school? [6 marks]

(c) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F? [4 marks]

(d) An highland has two kinds of inhabitants, knights and knaves. Knights always tell the truth, and only the truth; Knaves always tell lies, and only lies. John encountered two people on his visit to the highland, A and B. Determine what is A and B if A tells John " B is a Knight" and B "says The two of us are of opposite type"

[4 marks]

Question 4 [20 marks]

(a) Find the number M of seven letter words that can be formed using the word "BENZENE". [8 marks]

(b) Use Binomial theorem to Determine the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$

[4 marks]

(c) Determine the expansion of $(x+y)^4$ using Binomial theorem [4 marks]

(d) Discuss the pigeonhole principle and its applications [4 marks]

Question 5[20 marks]

Let *p*, *q*, and *r* be the propositions

p: You get an A on the final exam

q: You do every set of homework of this class

r: You get an A in this class

a) Write the following propositions using *p*, *q*, and *r* and logical connectives [6 marks]

i. To get an A in this class, it is necessary for you to get an A on the final.

ii. Getting an A on the final and doing every set of homework of this class is sufficient for getting an A in this class.

iii. You get an A on the final, but you don't do every set of homework of this class; nevertheless, you get an A in this class.

b. Translate the following propositions into English sentences. [6 marks]

i. $r \leftrightarrow p\ '\ q$

ii. $\neg(p \wedge q) \rightarrow \neg r$

iii. $\neg((p \wedge q) \rightarrow r)$

c. With the use of direct proof or otherwise, prove the following:

(i). The square of an even natural number is even [4 marks]

(ii). The square of an odd natural number is odd [4 marks]
