

CHUKA



UNIVERSITY

## UNIVERSITY EXAMINATIONS

### FIRST YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

#### ACTM 102: FUNDAMENTALS OF ACTUARIAL MATHEMATICS I

STREAMS: BSC (ACTR SCI)

TIME: 2 HOURS

DAY/DATE: MONDAY 10/12/2018

2.30 PM – 4.30 PM

#### INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

#### QUESTION ONE (30 MARKS)

a.) Find the survivorship function of  $l_x$  if force of mortality is given by

$$\mu_x = \frac{1}{100-x} \quad \text{for } x < 100 \quad [4 \text{ marks}]$$

b.) At a certain company, the probability of each employee leaving during any given year is 5%, independent of the other employees. Those who remain with the company for 25 years are given Ksh1, 000,000. What is the expected present value of this payment to a new starter, assuming an interest rate of 7% *pa* and ignoring the possibility of death? [3 marks]

c.) A population with limiting age 100 has the following survival function:

$${}_tP_0 = \left(1 - \frac{t}{100}\right)^{0.5} \text{ for } 0 \leq t \leq 100$$

Calculate the complete expectation of life at age 50. [6 marks]

d.) Explain why we study fundamentals of actuarial mathematics. [4 marks]

e.) Calculate:  $5|10q[52]$   
 Basis:  
 Mortality: AM92 Select [4 marks]

f.) Define the force of mortality and give its mathematical formula [3 marks]

g.) (i.) What is annuity? [2 marks]

(ii.) Explain two types of annuities you know [4 marks]

**QUESTION TWO (20 MARKS)**

a.) A population is subject to a constant force of mortality of 0.015.

Calculate:

(i.) The probability that a life aged 20 exact will die before age 21.25 exact. [5 marks]

(ii.) The curtate expectation of a life aged 20 exact. [5 marks]

b.) You are provided with the following extract from a life table:

$x$	$l_x$
50	99,813
51	97,702
52	95,046

Calculate  $0.75p_{50.5}$  using two different methods. [5 marks]

**QUESTION THREE (20 MARKS)**

a.) In a special mortality table with a select period of one year, the following relationships are true for all ages:

$$0.5q_{[x]} = (0.33)q_x$$

$$0.5q_{[x]+0.5} = (0.5)q_x$$

Express  $p[x]$  in terms of  $px$ . [10 marks]

- b.) (i.) Define and calculate the value of  $5|10 q[40]+1$ .  
 Basis: AM92 Select [7 marks]
- (ii.) Explain the meaning of differed probabilities [3 marks]

**QUESTION FOUR (20 MARKS)**

- a.) In a certain non-select mortality table that follows a uniform distribution of deaths (U.D.D.), find the values of the following in terms of  $l_x$  ;
- (i.)  $1.5P_{30.5}$  [5 marks]
- (ii.)  $\mu_{30.5}$  [5 marks]
- b.) State and explain the common laws of mortality [10 marks]

**QUESTION FIVE (20 MARKS)**

- (a) Fill in the table below (5 marks)

Age $x$	$l_x$	$d_x$	$p_x$	$q_x$
30	94726			
31			0.99839	
32	94425			0.00167
33		164		
34			0.99817	
35		182		

Using the table;

- (i.) What is the proportion aged 31 expected to live to age 35 [4 marks]
- (ii) What is the average number of persons who might be expected to die between age 31 and 35 out of 3000 persons aged 30 now [4 marks]
- (b) Using the actuarial life tables, what is the chance that a male child born to a mother aged 31 and a father aged 33 will live be alive 2 years but orphaned by both parents [7 marks]
- .....