ACMT 202

FUNDAMENTALS OF ACTUARIAL MATHS II

QUESTION ONE (30 MARKS)

a. A maintenance contract on a hotel promises to replace burned out light bulbs at the end of each year for three years. The hotel has 10,000 light bulbs; the light bulbs are all new. If a replacement bulb burns out, it too will be replaced with a new bulb.
You are given
   i. For a new bulb q_0= 0.10, q_1 = 0.30 and q_2 = 0.50
   ii. Each bulb cost Kshs. 1.
   iii. $i = 0.05$

Calculate the actuarial present value of the contract (5 marks)

b. Differentiate between a whole life insurance and level benefit insurance (2 marks)

c. What is an n-year pure endowment policy? Given a pure endowment if Ksh.1, issued to (x) with a term of n years. Deduce the present value and the expected present value (6 marks).

d. Suppose that the age-at death random variable is exponential with constant force of mortality μ Let $\overline{Z}^1/_{x:n\urcorner}$ be the present value of n-year endorsement for a life aged (x) with the benefit payment of 1. Assume the force of interest δ.
find
   i. $A^1/_{x:n\urcorner}$
   ii. ${}^2A^1/_{x:n\urcorner}$
   iii. Var $(\overline{Z}^1/_{x:n\urcorner}$

(6 marks)

e. in life insurance, what is the definition of recursion relations. Given two forms with their formulas f applications of recursion formulas (4 marks)

f. show that

$$A^1/_{x:n\urcorner} = Vqx+Vpx\ A\ ^1/_{x+i:n-1\urcorner} \qquad \text{(4 marks)}$$

g. List and explain applications of life insurance plans (3 marks)

QUESTION TWO (20 MARKS)

a. What is a whole life annuity? List and explain two types of whole life annuity (6 marks).
b. For a disability insurance claim
   i. The claimant will receive payments at a rate of Khs. 20,000 per year, payable continuously as long as she remains disabled.
   ii. The length a payment period in years is a random variable wit pdf $F(t) = te^{-t}$, $t>1$
   iii. Payments begin immediately.
   iv. $\delta = 0.05$

Calculate the actuarial present value of the disability payments at the time of disability (6 marks).

c. (i) Explain and define a continues n-year temporary life annuity and give its scenarios of payment (4 marks).
   (ii) Deduce its present value (2 marks)
   (iii) Deduce the actuarial present value (2 marks)

QUESTION THREE (20 MARKS)

a. List and explain three types of discrete life annuities (6 marks)
b. For a 5 year deferred whole life annuity due of 1 on (x), you are given
   i. $\mu(_{x+t}) = 0.01$
   ii. $i = 0.01$
   iii. $\ddot{a}_{x:5} = 4.542$

The random variable S donates the sum of annuity payments

   i. Calculate $_{5|}\ddot{a}_x$ (5 marks)
   ii. Calculate $Pr(s > {}_{5|}\ddot{a}_x$ (4 marks)

c. Consider a 5 year certain and life annuity dues for (60) that pays Kshs. 1,000 guaranteed at the beginning of the year for 5 year and counting thereafter for life. You are given the following:
   i. $i = 0.06$
   ii. $A_{65} = 0.43980$
   iii. $l_{60} = 8188$ and $l_{65} = 7534$

Calculate the actuarial present value of the annuity (5 marks)

QUESTION FOUR (20 MARKS)

a. What is an immediate n-year deferred annuity? Write down its present value and its actuarial present value. (5 marks)

b. The age at death random variable obeys De Moivre Law on the interval (O,W). Let $\overline{Z}_x$ be the contigent payment random variable for a life aged x. assume a constant force of interest δ. Find

   i. $\overline{A}_x$ (2 marks) iii) Var (Z_x) 2 marks

   ii. ${}_2\overline{A}_x$ (2 marks)

c. The lifetime of a group of people has the following survival function associated with it. $S_{(cx)} = 1 - {}^{x}/_{100}$, $0 \le x \le 100$.
   Frank, a member of the group is currently 40 years and has a 15-year endowment insurance policy, which will pay him Kshs. 50,000/= upon death. Find the actuarial present value of this policy. Assume an annual force of interest δ = 0.05 (5 marks)

d. List and explain four factors in product pricing (4 marks)

QUESTION FIVE (20 MARKS)

a. What is a surrender value in insurance? List and explain factors that should be considered in reaching for the minimum surrender value for policy holder (three explained points) (8 marks)

b. Show that $m|\overline{A}_x + \overline{A}^1/_{x:m} = \overline{A}_x$ (6 marks)

c. Let the remaining lifetime at birth random variable X be uniform on [0,100]. Let $({}_{10}|Z_{30})$ be the contingent payment random variable for a life aged x = 30.
   Find

   i. ${}_{10}|A_{30}$ 2 marks

   ii. ${}^{2}{}_{10}|A_{30}$ 2 marks

   iii. Var $({}_{10}|Z_{30})$ 2 marks

   If δ = 0.05