CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

ACMT 413: RISK MODELING

STREAMS: BSC **TIME: 2 HOURS**

DAY/DATE: THURSDAY 23/09/2021 **2.30 P.M – 4.30 P.M.**

INSTRUCTIONS:

QUESTION ONE (30 MARKS) COMPULSORY)

A) List and explain two major statistical paradigms and give two examples each. (6 marks)

B) What are the two assumptions that are based on Emperical Bayes Credibility Theory model I? (4 marks)

C) Write down formulae in terms of M_k for the mean, variance and coefficient of skewness of random variable x_i, representing individual claim sizes (6 marks)

D) State the Lundeberg's inequality (2 marks)

E) An annual aggregate claim amount from a risk has a compound poison distribution with posion parameter 10. Individual claim amounts are uniformly distributed on (0, 2000). The insurer of the risk has affected excess of loss reinsurance with retention level 1,600. Calculate

i) Mean (3 marks)

ii) Variance (3 marks)

iii) Coefficient of skeweness of both the insurer and reinsurer's aggregate claims under the reinsurance arrangement (3 marks)

What are the criteria that have to be met for a risk to be insurable? (3 marks)

QUESTION TWO (20 MARKS)

a) List an explain attributes of short term insurance contract (3 marks)

b) A compound distribution S is such that P (N=0) = 0.6, P (N=1) = 0.3 and P (N=2) =0.1 Claim amount are either for 1, unit or 2 units, each with probability 0.5. Derive the distribution function of S. (5 marks)

c) The table below shows the aggregate claim amounts in Kshs. *m*) for Jubilee insurance for a period of 5 years. Fill in the missing entries and calculate E [m(θ)] and Var [m(θ)] E(s^2 (θ)] using EBCT I model.

d)

TOTAL CLAIM AMOUNT

	Years(J)					x_1	$\frac{1}{4}\sum_{j=1}^{5}(x_{ij}-x_i)^2$
Country	1	2	3	4	5		
1	48	53	42	50	59	50.4	39.3
2	64	71	64	73	70	68.4	17.3
3	85	54	76	65	90	74.0	213.5
4	44	52	69	55	71	?	?

(12 marks)

QUESTION THREE (20 MARKS

a) Describe three situations where an insurer might determine a premium rate by combining data for a risk with collateral data. (3 marks)

b) A stastician wishes to find a Bayesian estimate of the mean of an exponential distribution with density function $f(x) = {}^{1}/\mu\ e^{-x}/\mu$. he is proposing to use a prior distribution of the form

$$\text{Prior } (\mu) = \frac{\theta \propto e^{-\theta/\mu}}{\mu^{\alpha+1}\sqrt{\propto}} \quad \mu>0$$

You are given the mean to be $\theta / \propto_{-1}$

i) Write down the likehood function for μ based on the random samples of value x_1, $x_2 \ldots x_n$ from the exponential distribution. (2 marks)

ii) Find the form of the posterior distribution for μ and hence show that on expression for the Bayesian estimate for μ

$$\mu = \frac{\theta+\Sigma x_2}{n+\propto-1}$$ (5 marks)

iii) Show that the Bayesian estimate for μ can be written in the form of credibility estimate. Write down the formula for credibility factor (5 marks)

iv) The statistician now decides that he will use a prior distribution of this form with parameters θ=40 and ∝=1.5. His sample data have statistics n=100, Σx= 9,826 and Σx_1^2= 1,200,000. Find the posterior estimate for u and the value of credibility factor. (5 marks)

QUESTION FOUR (20 MARKS)

a) If X~ poison (d) and y~ poison (u) are independent random variables. Fin the probability function of Z=x+y. using convolutions. (4 marks)

b) Find the expression of MGF (Moment Generating Function) of a claim amount if the number of claims has a bin (100, 0.01) distribution and the individual claim size are gamma (10,0.2). Find the mean and variance of the aggregate claim amount. (6 marks)

c) What is proportional reinsurance. (2 marks)

d) List and explain approaches to credibility theory. (8 marks)

QUESTION FIVE (20 MARKS)

A) The aggregate claims from a risk have a compound poison distribution with parameter μ. individual claim amount (Kshs.) have a pareto distribution with parameter ∝=3 and λ=1,000. The insurer of the risk calculates the premium using a premium loading factor of 0.02. The insurer is considering effecting excess of loss reinsurance with retention limit of

Kshs. 1,000. The reinsurance premium will be calculated using a premium loading factor of 0.3. the insurer profit is defined to be the premium charged by the insurer less than reinsurance premium and less the claims paid by the insurer, net of reinsurance,

i. Show that the insurer expected profit before reinsurance is 100μ.

ii. Calculate the insurers expected profit after affecting the reinsurance, and hence find the percentage reduction in the insurer expected profit.

iii. Calculate the percentage reduction of the standard deviation of the insurer profit or a result of affecting reinsurance. (20 marks)
