CHUKA  **UNIVERSITY**

UNIVERSITY EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 452: HYPOTHESIS TESTING

STREAMS: BSC MATHEMATICS, BSC ECON STAT; BA ECON, MATH; BED SCI
TIME: 2 HOURS

DAY/DATE : WEDNESDAY 22 /09/ 2021 **2.30 PM – 4.30 PM**

INSTRUCTIONS TO CANDIDATES:

1. Time 2 Hours
2. Answer **Question ONE (Compulsory)** and any other **TWO** Questions
3. Do not write on the question paper

QUESTION ONE – COMPULSORY (30 MARKS)

(a) (i) Explain the term "Statistical Hypothesis"

(ii) State the major steps involved in solving a test of hypothesis problem. [6 marks]

(b) Differentiate between the following terms as used in statistical inference:
(i) Simple and composite statistical hypotheses. Give examples.

(ii) Randomized and Nonrandomized tests. [6 marks]

(c) Explain the following terms:
(i) Type I and II errors [4 marks]
(ii) Power function of a test [2 marks]

(d) Explain the theory of sequential likelihood ratio testing. [6 marks]

(e) Juma performed an experiment four times where the probability of success was β. He set the hypothesis as: $H_0 : \beta = \frac{2}{5}$ against $H_1 : \beta = 0.25$. The null hypothesis was rejected whenever the number of successes was less than two. Determine:

i) The probability of type I error
ii) Probability of type II error
iii) The power of the test when $\beta = \frac{1}{3}$ [6 marks]

QUESTION TWO (20 MARKS)

a) The data below represents a sample of mathematics achievement test scores and calculus grades for 10 independently selected college students. From evidence, would you say that the achievement test scores and calculus grades are different? Take $\alpha = 0.05$.

Student	1	2	3	4	5	6	7	8	9	10
Math test score (X)	39	43	21	64	57	47	28	75	34	52
Final calculus grade (Y)	65	78	52	82	92	89	73	98	56	75

[10 marks]

b) The mean birth weight in CHUKA hospital is $\mu = 3315$ grams, with standard deviation of $\sigma = 575$ grams. Let X denotes the birth weight in grams in CHUKA hospital. If X is assumed to be normally distributed with mean μ and unknown variance σ^2.

Required

i. Formulate a statistical hypothesis for a researcher who is studying a sample of 30 babies of MERU hospital that the mean birth weight of these children is greater than that of CHUKA hospital. [1 marks]
ii. Derive the BCR for this test at $\alpha = 0.05$. [5 marks]
iii. If this random sample of size n = 30 yielded $\bar{x} = 3189$ and s = 488, what conclusions do you make? [4 marks]

QUESTION THREE (20 MARKS)

(a) Let X be a normal random variable with unknown mean μ and variance 1 and having a sample size of 100. The hypothesis is set as: H_0: μ = 1 against H_1: μ = 2.
Obtain a test:

i) that minimizes probability of Type I error
ii) that minimizes probability of Type II error
iii) That is most powerful when μ = 2 at α = 0.05 [15 marks]

(b) In comparing the variability of the tensile strength of two kinds of structural steel, an experiment yielded the following results: m=6, n=9. The sample variance of the first sample was 19.2 and for the second sample was 3.5. Assuming the measurements constitute an independent sample from normal population. Using $\alpha = 0.05$,test the null hypothesis that $H_0: {\sigma_1}^2 = {\sigma_2}^2$ vs $H_1: {\sigma_1}^2 \neq {\sigma_2}^2$ [5 marks]

QUESTION FOUR (20 MARKS)

(a) A random sample $X_1, X_2, \ldots, X_n$ is obtained from a Bernoulli distribution $f(x,p) = p^x (1 - p)^{1-x}$ With an unknown parameter p. Find the uniformly most powerful size α test for the hypothesis $H_0 : p = p_0$ against $H_1 : p > p_0$ [10 marks]

(b) Define the likelihood ratio tests and state its properties. [10 marks]

QUESTION FIVE (20 MARKS)

(a) State and proof the Neyman-Pearson lemma for a simple hypothesis against a simple alternative. [15 marks]

(b) A random sample of 19 babies of a certain age was taken from a normal population and it was found that the standard deviation of their weights is 2.5 kg. Test the hypothesis $H_0: \sigma_0 = 3$ Versus $H_1: \sigma_0 \neq 3$ at $\alpha = 5\%$. [5 marks]

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