CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

MATH 206: INTRODUCTION TO REAL ANALYSIS

STREAMS: "AS ABOVE" Y2S2 **TIME: 2 HOURS**

DAY/DATE: TUESDAY 02/11/2021 **11.30 A.M – 1.30 P.M.**

INSTRUCTIONS:

- Answer question **ALL** the questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Prove that a limit of function exists then that limit is unique (5 marks)

(b) Define an open subset A of ℝ. Hence determine whether the set $A = \{x \in \mathbb{R}: 2 \leq x < 5\}$ is open or not in ℝ. (5 marks)

(c) Given the set $A = \{x \in \mathbb{R}: a \leq x < b\}$. Determine if possible, the lower boundary, infimum, upper boundary and supremum of the set. (5 marks)

(d) Briefly describe the Riemann Integrable function f on the interval $[a, b]$ (5 marks)

QUESTION TWO: (20 MARKS)

(a) Let $a\ and\ b$ be non-negative real numbers. Prove that

(i) there exist always a non-negative real number a^{-1} (3 marks)

(ii) $a < b$ if and only if $a^2 < b^2$ (4 marks)

(b) Given that $A \subseteq \mathbb{R}$, define an interior point x of A. Hence show that if A is open if and only if A is equal to its interior set A^0 (5 marks)

(c) Using the definition of limit of a function, prove that

(i) $\lim_{n\to\infty}\left(\frac{(-1)^n}{n+5}\right) = 0$ (4 marks)

(ii) $\lim_{x\to 2}(x^3 + x - 10) = 0$ (4 marks)

QUESTION THREE: (20 MARKS)

(a) Prove that if a function is differentiable at a point $x = a$ then the function is also continuous at the same point. (5 marks)

(b) Hence show that the function $f(x) = 2x$ is Riemann Integrable on the interval $[0,1]$

(8 marks)

(a) Find the limit superior and limit inferior of the sequence

$X_n = \left(1 + \frac{n}{n+1} + cos\ \frac{n\pi}{2}\right) : n \in \mathbf{N}$ (7 marks)
