

CHUKA

UNIVERSITY



UNIVERSITY EXAMINATIONS

**SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION
SCIENCE/ARTS, BACHELORS OF SCIENCE, BACHELORS IN MATHEMATICS,
BACHELORS OF ARTS(MATHS-ECONS), BACHELORS OF SCIENCE(ECON STATS)**

MATH 222: VECTOR ANALYSIS

JAN-MARCH 2021 SESSION

STREAMS: `` As above``

TIME: 2HRS

DAY/DATE:
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INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (30 MARKS)

(a) Prove that if \vec{a} and \vec{b} are non-collinear vectors and $x_1\vec{a} + y_1\vec{b} = x_2\vec{a} + y_2\vec{b}$, then $x_1 = x_2$
and $y_1 = y_2$ (3 marks)

(b) The position vector of a particle in space is given by the equation $\vec{s} = 2 \cos 2t \hat{i} + 2 \sin 2t \hat{j} + 3t\hat{k}$

(i) Find the equations of the velocity and acceleration at any time t (2 marks)

(ii) Find the initial speed of the particle (2 marks)

- (c) Find the directional derivative of $\phi = x^2y + xz$ in the direction of the vectors $2\hat{i} - 2\hat{j} + \hat{k}$ at $(1, -2, 2)$ (4 marks)
- (d) Show that $\nabla^2 \left(\frac{1}{r}\right) = 0$ (4 marks)
- (e) Find the work done in moving a particle once around a circle C in the x-y plane if the circle has centre at the origin and radius 3 units and the force field is given by

$$\vec{F} = (2x - y + z)\hat{i} + (x + y - z^3)\hat{j} + (x - 6y + z)\hat{k}$$
 (4 marks)
- (f) Find an equation for the plane determined by the points P1(2,-1,1), P2(-3,-2,1) and P3(-1,3,2) (4 marks)
- (g) Given that $\phi = 45x^2y$, evaluate its volume integral such that the volume space is bounded by planes $4x + 2y + z = 8$, $x = 0$, $y = 0$, $z = 0$. (5 marks)
- (h) If $\vec{A} = (2x^2y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + (x^2\cos y)\hat{k}$, find $\frac{\delta^2 A}{\delta x \delta y}$ (2 marks)

QUESTION TWO: (20 MARKS)

- (a) Given that $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = \hat{i} - 2\hat{j} - 4\hat{k}$ are position vectors of points P and Q respectively. Find an equation for the plane passing through Q and perpendicular to the line PQ. (6 marks)
- (b) Given the space curve defined by $\vec{r}(t) = \langle t, 3\sin t, 3\cos t \rangle$, Find
 (i) The tangent vector \vec{T} (3 marks)
 (ii) The principal normal \vec{N} and curvature κ (3 marks)
 (iii) The Binormal \vec{B} (3 marks)
- (c) Prove that $\text{div}(\text{Curl}(\vec{A})) = 0$ (5 marks)

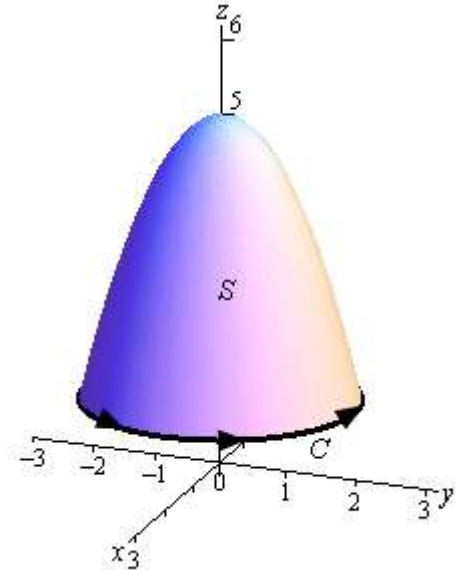
QUESTION THREE: (20 MARKS)

- (a) Given the vector function $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$
 (a) Show that \vec{F} is a conservative force field (3 marks)
 (ii) Find the scalar potential (4 marks)
- (iii) Evaluate $\int_c \vec{F} \cdot d\vec{r}$, where c is a straight line from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$ (3 marks)
- (b) State and Verify divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 4$, $0 \leq z \leq 3$ (10 marks)

QUESTION FOUR: (20 MARKS)

(a) Evaluate the line integral $\int_C y^3 dx - x^3 dy$ where C is the positively oriented circle of radius 2 centered at the origin. (10 marks)

(b) Use Stoke's theorem to evaluate $\iint_S \text{curl } \vec{F} \cdot \vec{ds}$, where $\vec{F} = z^2 \vec{i} - 3xy \vec{j} + x^3 y^3 \vec{k}$ and S is the part of surface $z = 5 - x^2 - y^2$ above the plane $z = 1$. Assume that S is oriented upwards.



(10 marks)

QUESTION FIVE: (20 MARKS)

(a) Find the volume of the parallelepiped with adjacent sides

$$\vec{u} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{v} = -\hat{i} + 3\hat{j} + 2\hat{k}, \vec{w} = \hat{i} + \hat{j} - 2\hat{k} \quad (4 \text{ marks})$$

(b) If $\vec{A} = x^2 y z \hat{i} - 2xz^3 \hat{j} + xz^2 \hat{k}$ and $\vec{B} = 2z \hat{i} + y \hat{j} - x^2 \hat{k}$, find $\frac{\partial^2}{\partial x \partial y} (\vec{A} \times \vec{B})$ at (1,0,-2)

(5 marks)

(c) Verify the Green's Theorem for $\int_C x^3 dx + 2x(1 + xy) dy$, where C is the unit circle.

(11 marks)

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