CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE/ARTS, BACHELORS OF SCIENCE IN MATHEMATICS, BACHELORS OF ARTS (MATHS-ECONS)

(MAY-JULY 2021)

MATH 303: REAL ANALYSIS II

STREAMS: ````as above```` Y3S2 **TIME: 2HRS**

DAY/DATE: ……………………. ………………………..

INSTRUCTIONS:

- Answer question **ONE** and **TWO** other questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE: (20 MARKS)

(a) Let f_n be convergent sequence of real valued functions whose limit is f, prove that if $c \in \mathbb{R}$ then (cf_n) is convergent to the limit cf (4 marks)

(b) State and prove the change of basis property for logarithms of numbers (4 marks)

(c) By sketching the graphs of the function $f(x) = log_a x$ for values of $a = 3$ and $a = \frac{1}{3}$ on the same axis, state the relationship between the two graphs (4 marks)

(d) State without proof the D'Alembert RatioTest for convergence of infinite series of functions (4 marks)

(e) Illustrate that all Dirichlet functions are Characteristic functions but the converse is not true (4 marks)

(f) Distinguish between an absolutely convergent and conditionally convergent series

(2 marks)

(g) Let $f(x) = 4x + 1 \quad for \;\; 0 \leq x \leq 1$ and $P = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$. Find The Riemann's' upper sum $U(P, f)$ of the function f

(4 marks)

(h) Define a Step function. Hence show that a step function is always Riemann Integrable

(4 marks)

QUESTION TWO: (20 MARKS)

(a) (i) State and prove the Comparison test (Weiertrass M-Test) for convergence of series of real valued functions

(8 marks)

(ii) Hence Using the Comparison test show that the series $\sum_{n \in N} \frac{1}{n^P}$ is divergent for $p < 1$

(5 marks)

(b) (i) State and prove the Intermediate Mean Value Theorem (5 marks)

(ii) Hence use it to show that the $f(x) = x^3 - 2x^2 + 2x - 4$ has a zero in the interval $[0, 3]$

(2 marks)

QUESTION THREE: (20 MARKS)

(a) Describe how the area under a curve can be obtained using the Riemann-Stieltjes Integration method

(5 marks)

(b) Show that the function $f(x) = 3x$ is Riemann Integrable on $[0,1]$ and that $\int_0^1 f(x) = 1.5$

(10 marks)

(c)) Let $\sum_{n \in N} f_n$ be a series of functions on **K.** Prove that this series only converges if

$$\forall \, \varepsilon > 0 \; \exists \; N(\varepsilon) \in \boldsymbol{N}: \left|\sum_{k=m}^{n} f_n\right| < \varepsilon \;\; for \; every \; n \geq m \geq N(\varepsilon)$$

(5 marks)

QUESTION FOUR: (20 MARKS)

(a) State and prove the Cauchy's Root Test for convergence of functions of an infinite series.

(13 marks)

(b) Prove that an absolute convergent series of functions in $(\boldsymbol{K}, d)$ is necessarily convergent, however by use of an appropriate counter example show that the converse not true.

(7 marks)

QUESTION FIVE: (20 MARKS)

(a) Derive the Fourier coefficients of the function $f(x)$ over the integral interval of $-l$ to l

(10 marks)

(b) Hence find the Fourier series of the function defined by

$$f(x) = x, \quad for \; -\pi \leq x < \pi$$

(10 marks)
