CHUKA UNIVERSITY

UNIVERSITY EXAMINATION

RESIT /SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE

MATH 313/303: REAL ANALYSIS II

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 04/11/2021 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS:

- **Answer question ALL the questions**
- **Sketch maps and diagrams may be used whenever they help to illustrate your answer**
- **Do not write on the question paper**
- **Write your answers legibly and use your time wisely**

QUESTION ONE: (30 MARKS)

(a) (i) When is the sequence x_n of elements of real or complex numbers said to be convergent? (2 marks)

(ii) Let x_n, y_n and z_n be sequences of real numbers such that $x_n \leq z_n \leq y_n \quad \forall\, n \geq N$(N is a fixed integer). Let x_n, y_n both converge to the same limit, say $\boldsymbol{l}$**.** Show that z_n also converges to $\boldsymbol{l}$ $as\ n \to \infty$ (5 marks)

(b) Take $a, b > 0$ $(a, b \neq 1)$, prove that $log_a x = \frac{log_b x}{log_b a}$ (3 marks)

© Define and give an example of a periodic function (2 marks)

(d) (i) Define an absolutely convergent series (2 marks)

(ii) Show that in general absolute convergence implies convergence in (K, d) (3 marks)

(e) (i) Let $\sum_{k\in N} x_k$ be a series of real numbers. Prove that if $|x_k| \leq y_k\ \forall\ k \in \boldsymbol{N}$ and $\sum_{k\in N} y_k$ is convergent, then the sum $\sum_{k\in N} x_k$ is also convergent (4 marks)

(ii) Prove that if $p = 1$, then the series $\sum_{n \in N} \frac{1}{n^P}$ is divergent (5 marks)

(f) Define the Fourier series of the function $f(x)$ on the interval $-l \ to \ l$ (4 marks)

QUESTION TWO: (20 MARKS)

(a) (i) Write the general expression of an exponential and logarithmic function whose base is a (2marks)

(ii) By considering $a > 1 \ and \ 0 < a < 1$ for the functions f(x) and g(x) respectively make a comparison of the exponential and logarithmic functions. Hence state any three differences in these graphs. (8marks)

(b) Find the Fourier series of the function defined by

$$f(x) = 0, \quad for - \pi < x < 0, \quad and \qquad f(x) = x \quad for \ 0 < x < \pi$$

(10marks)

QUESTION THREE: (20 MARKS)

(a) (i) Describe the Riemann Integrable function f on the interval $[a, b]$ (4 marks)

(ii) Show that a Dirichlet function on the interval $[a, b]$ is not Riemann Integrable. (6 marks)

(b) Show that the function $f(x) = x$ is Riemann Integrable in $[0,1]$ that $\int_0^1 f(x) = \frac{1}{2}$ (10 marks)
