CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

RESIT/SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE/ARTS

MATH 400/401: TOPOLOGY 1

STREAMS: BSC/ARTS **TIME: 2 HOURS**

DAY/DATE: THURSDAY 12/08/2021 **2.30 P.M – 4.30 P.M.**

INSTRUCTIONS
Answer ALL the three questions.
Do not write on the question paper.

QUESTION ONE: (30 MARKS)

(a) Distinguish the following terms as used in topology

(i) An indiscrete topology and a discrete topology

(ii) A dense subset and a nowhere dense subset

(iii) The interior of the point p and a neighborhood of the point p

(iv) A base and a subbase for the topology τ

(v) $a\ T_1\ and\ T_2$ space (10 marks)

(b) (i) Prove the general De Morgan's law for the intersection;

$X\backslash\cap_{A_\lambda} = \cup\ (X\backslash A_\lambda)\ for\ \lambda\ \epsilon\ \Lambda$ (4 marks)

(ii) Prove that finite union of closed sets is also closed. (3 marks)

(c) (i) Define a limit point of a subset A of a topological space X. (1 mark)

(ii) Let $X = \{a, b, c, d, f\}$ and $\tau = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{c\}, \{a, f\}, \{f\}, \{b, c, f\}, \{a, b, c, f\}\}$. Let $A = \{a, c, d\}$. Show that b is a limit point of A but a is not. (6 marks)

(d) Let $f: x_1 \rightarrow x_2$ $where\ x_1 = x_2 = \{0,1\}$ and are such that $(x_1, D) and\ (x_2, \$)$ be defined by $f(1) = 1\quad and\ f(0) = 0$. Show that f is not continous but f^{-1} is continuous. (4 marks)

(e) Define the term a topological property (p). Give an example for this property. (2 marks)

QUESTION TWO: (20 MARKS)

(a) Let $p \in X$ and denote N_P the set of all neighborhood of a point p. Prove that the following

(i) $\forall\ pairs\ N, M \in N_P,\ \ N \cap M \in N_P$ (3 marks)

(ii) If $N \in N_P$ and for every $M \subset X$ with $N \subset M$ it implies that $M \in N_P$

(3 marks)

(b) Let (X, τ) be a topological space and $A, B \subset X$. Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$ (3mks)

(c) Let (X, τ) be a topological space and $A, B \subset X$. Denote A^0 the interior of A.

(i) Using an appropriate example, show that $A^0 \cup B^0 \neq (A \cup B)^0$ (3 marks)

(ii) Prove that $A^0 \cap B^0 = (A \cap B)^0$ (4 marks)

(d) Distinguish a regular space and a normal space. (4 marks)

QUESTION THREE: (20 MARKS)

(a) Consider the following topology on $X = \{a, b, c, d, e\}$ and $\tau = \{\{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}, \{a, b, e\}, X, \emptyset\}$. If $A = \{a, b, c\}$. Find

(i) The exterior of A (3 marks)

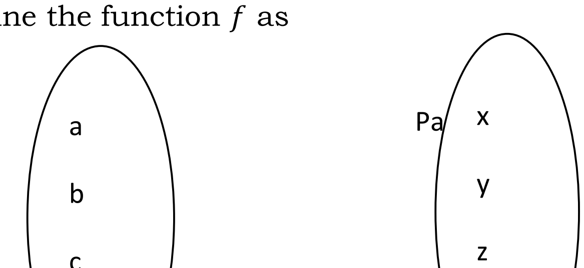
(ii) The boundary of A (2 marks)

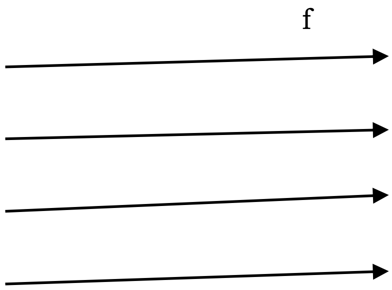
(iii) Hence show that the boundary of A, $\delta A = \bar{A} \cap \overline{X/A}$ (3 marks)

(b) Let $X = \{a, b, c, d\}$ $with\ \tau_X = \{\{a, b\}, \{a\}, \{b\}, X, \emptyset\}$ and Let $Y = \{x, y, z, t\}$ $with$

$\tau_Y = \{\{x\}, \{y\}, \{x, y\}, Y, \emptyset\}$.

Define the function f as





Show that the function f is a homomorphism. (5 marks)

(c) Distinguish between a $a\, T_1$ and T_2 space. Using an appropriate counter example show that a T_2 space $\Rightarrow T_1$ space but a T_1 space $\nRightarrow T_2$.

(7 marks)
