CHUKA UNIVERSITY

UNIVERSITY EXAMINATION

RESIT /SPECIAL EXAMINATION

EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF EDUCATION (SCIENCE, ARTS) BACHELOR OF SCIENCE

MATH 411/409: FUNCTIONAL ANALYSIS

STREAMS: **TIME: 2 HOURS**

DAY/DATE: THURSDAY 04/11/2021 **8.30 A.M – 10.30 A.M**

INSTRUCTIONS:

- Answer question **ALL** questions
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room

QUESTION ONE: (30 MARKS)

(a) In each of the following give reasons whether the relations given are functions or not

(i) $f = \{(7,2),(7,5)\}$

(ii) $h = \{(2,6),(1,,6)\}$ (4 marks)

(b) Distinguish an operator and a functional mapping on a vector space X (2 marks)

(c) Distinguish a meager subset and a non-meager subset of a metric space X. Hence state without proof, Baire's Category Theorem (4 marks)

(d) State the Parallelogram law as used in inner product spaces. Hence using an appropriate example, illustrate that all Banach spaces are not necessarily inner product spaces. (6 marks)

(e) Let $T: X \to Y$ be a linear operator from a normed linear space X into a normed linear space Y. Prove that if T is continuous at the origin it implies that T is uniformly continuous on X (4 marks)

(f) (i) Distinguish strongly convergence and weak convergence of sequences in a normed linear space X (2 marks)

(ii) Hence prove that the weak limit of a weakly convergent sequence is unique (4 marks)

(g) Distinguish an algebraic dual and a continuous dual (or simply the dual) of a linear space X (4 marks)

QUESTION TWO: (20 MARKS)

(a) Define a fixed point of a mapping T of a set X. Give two cases that illustrate a fixed point mapping. (3 marks)

(b) Prove that an orthonormal set is linearly independent (5 marks)

(c) Define a norm on a linear space X. Hence show that the mapping $\|.\|: R^n \to R$ defined by $\| x \|_\infty = \left((\sum_{k=1}^{n} \| x_k \|^2) \right)^{\frac{1}{2}}$ is a normed space. (12 marks)

QUESTION THREE: (20 MARKS)

(a) Prove that on the space of all sequences S, the mapping defined by

(i) $P_1(x) = \sup|x_n| \; \forall \, n \geq 1$ is a semi-norm (5 marks)

(ii) $p(x) = |x| = \sum_{k=1}^{\infty} \frac{1}{2^k} \frac{|x_k|}{1+|x_k|}$ is a paranorm (5 marks)

(b) Prove that strong convergence implies weak convergence, and with an appropriate counter example show that the converse is not necessarily true (10 marks)
