CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE AWARD OF DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS

MATH 204: ALGEBRAIC STRUCTURES

STREAMS: AS ABOVE **TIME: 2 HOURS**

DAY/DATE: WEDNESDAY 5/12/2018 **11.30 A.M - 1.30 A.M.**

INSTRUCTIONS:

- Answer Question **ONE** and any other **TWO** Questions
- Sketch maps and diagrams may be used whenever they help to illustrate your answer
- Do not write anything on the question paper
- This is a **closed book exam**, No reference materials are allowed in the examination room
- There will be **No** use of mobile phones or any other unauthorized materials
- Write your answers legibly and use your time wisely

QUESTION ONE (30 MARKS)

a) Define the following terms

i. A normal subgroup of a group G (1 mark)

ii. The kernel of homomorphism (1 mark)

iii. A cyclic group (1 mark)

iv. An ideal of a ring R (1marks)

b) Let $x * y = x + y - 5$ be a binary on the set of integers. Find the identity element and the inverse of 10001 (3 marks)

c) Given that $\phi : G \to G'$ is a homomorphism of groups, prove that the kernel of ϕ is a normal subgroup of G (5 marks)

d) Prove that every cyclic group is abelian (3 marks)

e) Let G be a group and $a,b,c \in G$, show that $a \circ b = a \circ c$ imply $b = c$ (3 marks)

f) The addition and part of the multiplication table for the ring R={a,b,c} are given below. Use the distributive laws to complete the multiplication table below (3 marks)

+	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

*	a	b	c
a	a	a	a
b	a	c	
c	a		

g) Let R be the ring of all 2X2 matrices over Z with the usual addition and multiplication of matrices.

i. Show that the subset of R consisting of all matrices of theform

$$T = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| a,b,c \in Z \right\}$$

is a non-commutativesubring with unity. (4 marks)

ii. Which elements of T are invertible? (2 marks)

iii. Find if $I = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a,b \in Z \right\}$ is an ideal of T (3 marks)

QUESTION TWO (20 MARKS)

a) Identify which of the following maps are group homomorphism and if it is, find its kernel

i. G is the group of non-zero real numbers under multiplication and $\varphi(x) = 2^x \, \forall x \in G$

ii. G is the group of real numbers under addition and $\phi(x) = x + 1$ (5 marks)

b) If $\alpha=(714), \beta=(4123), and \sigma=(34126)$ in S_7. Compute $\alpha\sigma^{-1}\beta$ as a single permutation. (5 marks)

c) Given that S ={All real numbers}. Define $a*b=\frac{a+b}{1+ab}$. Is ¿ associative?

(5 marks)

d) Show that the set $S=\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix},\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix},\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\right\}$ forms a cyclic group under matrix multiplication. (5 marks)

QUESTION THREE (20 MARKS)

a) Prove that a group G is abelianiff $(ab)^{-1}=a^{-1}b^{-1}\ \forall a,b\in G$ (4 marks)

b) Given the set $A=\{5,15,25,35\}$

i. Show that A is a group under multiplication modulo 40 (4 marks)

ii. Find the identity element (2 marks)

iii. Show that this group is isomorphic to the group formed by invertible elements in Z_8 (4 marks)

c) Consider the group $D_3 =< a,b : a^2 = b^3 = e; ba = ab^2$ > and the subgroup $H = \{e,b,b^2\}$.

i. List the right and left cosets of H in D_3 (5 marks)

ii. Is H a normal subgroup of D_3? (1 marks)

QUESTION FOUR (20 MARKS)

a) Show that if R is a ring and $a,b \in R$,where a is not a zero divisor, then $ba=1$ (4 marks)

b) Let R be a ring such that every element satisfiesthe equation $x^2 = x$, provethat Ris commutative (5 marks)

c) Let X be a non-empty set and Rbe the setoff all subsetsof X. define addition and multiplication in R as follows

$$A + B = A \cup B - A \cap B$$
$$A * B = A \cap B$$

For all $A \in R$ define a function $f : R \to Z_2$ as $f(x) = \begin{cases} 1 if x \in A \\ 0 otherwise \end{cases}$

i. Show that $A + \phi = A$ and $A + A = \phi$ (4 marks)

ii. Show that f is a homomorphism of rings (7marks)

QUESTION FIVE (20 MARKS)

a) Construct the multiplication table for the group of symmetries of a square (10 marks)

b) List all its subgroups (4 marks)

c) Show that the subgroup given by $H = \{e, b^2\}$ is a normal subgroup of D_4 (6 marks)